LM05 · DERIVATIVES · CFA LEVEL I

Pricing & Valuation of Forward Contracts

Initiation, during-life, and expiration values — plus forward rates for interest-rate forwards

What This Reading Covers

At initiation, $\mathbf{V_0(T) = 0}$ for a forward. The *price* $F_0(T)$ is locked, but the mark-to-market *value* $V_t(T)$ changes as time passes, rates move, or the underlying shifts. We also derive implied forward rates (IFRs) from spot zero rates, and price/value FRAs.

Learning Outcome Statements

LOS a Explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration

LOS b Explain how forward rates are determined for interest rate forward contracts and describe the uses of these forward rates

Exam Tip: Core formulas: $F_0(T) = S_0(1+r)^T$; $V_t(T) = S_t - F_0(T)/(1+r)^{T-t}$; $IFR_{A,B-A}$: $(1+z_A)^A(1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$.

LOS A · CORE DISTINCTION

Price $F_0(T)$ vs Value $V_t(T)$

At initiation (t=0)

$$V_0(T) = 0 \; for all forward contracts$$
$$F_0(T) = S_0(1+r)^T \; or \; S_0 e^{rT}$$
$$With carry: F_0(T) = [S_0 + \mathrm{PV}(c) - \mathrm{PV}(I)](1+r)^T \; or \; S_0 e^{(r+c-i)T}$$

At expiration (t=T)

$$V_T(T) = S_T - F_0(T)$$

Long forward gain = spot at T minus the locked-in forward price.

During the life (0 < t < T)

$$V_t(T) = S_t - F_0(T)/(1+r)^{T-t}$$

With cash flows: $V_t(T) = [S_t - PV_t(I) + PV_t(c)] - F_0(T) / (1+r)^{T-t}$

Three drivers of $V_t(T)$: (1) time passes, (2) interest rates change, (3) underlying price changes. The locked-in $F_0(T)$ does not move.

LOS A · MM PAGE 2

How $V_t(T)$ Moves — Decomposition Example

WORKED EXAMPLE **50-Stock Forward — Rate Change Alone**

GIVEN

$S_0 = 50$; $r = 2\%$; $T = 6$ mos.

INITIATION

$F_0(T) = 50(1.02)^{0.5} = \mathbf{50.50}$, $V_0(T) = 0$.

RATE RISES 20 BPS (R → 2.2%), T STILL 0

$V_0(T) = 50 - 50.50/(1.022)^{0.5} = 50 - 49.95 = \mathbf{0.05}$

T = 2 MOS., R_{120} = 1.6%, S_T = 50.25

$V_t(T) = 50.25 - 50.50 / (1.016)^{4/12} = 50.25 - 50.23 = \mathbf{0.02}$

Underlying +0.25; time + lower r takes back −0.23 → net 0.02.

Exam Tip: Value decomposition: long forward gains when S_t rises, and loses as time passes (carry unwinds) and when r falls.

LOS A · CARRY EXAMPLE

Forward Value with Income & Costs

WORKED EXAMPLE **Asset with Income I and Cost c — MM Page 3**

GIVEN

S_0 = 60. Income I = 0.50 at 3m and 6m. Cost c = 0.25 at 5m and 7m. T = 9m. Rates: r(90)=2%, r(120)=2.1%, r(180)=2.2%, r(210)=2.3%, r(270)=2.4%.

F_0(T) AT T=0

$F_0(T) = [60 - 0.5/(1.02)^{0.25} - 0.5/(1.022)^{0.5} + 0.25/(1.021)^{4/12} + 0.25/(1.023)^{7/12}] \times (1.024)^{9/12}$

$= (60 - 0.49753 - 0.49458 + 0.24827 + 0.2467) \times (1.024)^{9/12} = \mathbf{60.57}$

V_T(T) AT T=6M, S_T = 61.60

r(30)=1.6%, r(60)=1.7%, r(120)=2%. One remaining income at 1m, one remaining cost at 2m, discount F_0(T) over 4m:

$V_t(T) = (61.60 - 0.5/(1.016)^{1/12} + 0.25/(1.017)^{2/12}) - 60.57/(1.02)^{4/12}$

$= (61.60 - 0.49934 + 0.249298) - 60.1715 = \mathbf{1.18}$

LOS A · COVERED INTEREST PARITY

FX Forwards — Price & Value

$$F_0^{P/B} = S_0^{P/B} \cdot e^{(r_P - r_B)T} \qquad V_0(T) = 0 at initiation$$

A widening ($r_P - r_B$) supports gains on a *long base* position. Long gains if r_P ↑, r_B ↓, or $S^{P/B}$ ↑.

WORKED EXAMPLE **Long USD/EUR Forward at 1.2010**

GIVEN

$S_{USD/EUR} = 1.192$; $r_{USD} = 0.5\%$; $r_{EUR} = -0.25\%$; T = 1 yr; long at 1.2010.

VALUE AFTER R_{USD} RISES 25 BPS (TO 0.75%)

$V_t(T) = 1.192 - 1.2010 \cdot e^{-(0.0075 - (-0.0025))} =$ **0.00295**

× €50M notional = **147,500 USD** gain.

WORKED EXAMPLE **Short 6-mo ZAR/EUR at 17.2506**

GIVEN

At t=0: $S_{ZAR/EUR} = 16.909$; $r_{ZAR}(180)=3.5\%$, $r_{EUR}(180)=-0.5\%$. After 2m: $S_t = 17.02$; $r_{ZAR}(120)=3\%$, $r_{EUR}(120)=-0.3\%$.

VALUE OF SHORT POSITION

$V_t(T) = [F_0(T) \cdot e^{-(r_P - r_B)(T-t)}] - S_t = 17.2506 \cdot e^{-(0.03-(-0.003)) \cdot 4/12} - 17.02 =$ **0.01488** × €50M = **2,094,162.26 ZAR**.

QUIZ 1 · PRICE & VALUE

Initiation, Life & Expiration

Q1. The value of a forward contract at initiation is:

Q2. S_0=50, r=2%, T=6 mos. Two months later, S_t=50.25 and r_{120}=1.6%. $V_t(T)$ is closest to:

Q3. At expiration, the value of a long forward is:

Q4. A long USD/EUR forward at 1.2010, with S_0=1.192, r_{USD}=0.5% and r_{EUR}=−0.25%, T=1yr. If r_{USD} rises 25 bps, $V_t(T)$ per EUR is closest to:

Q5. The three drivers of $V_t(T)$ during the life of a forward are:

LOS B · TERM STRUCTURE

Zero Rates, Discount Factors & Forward Rates

The forward rate model

$$(1 + z_A)^A \cdot (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$$

Notation: $\mathbf{IFR_{x,y}}$ — implied forward rate starting in *x* periods for *y* periods.

WORKED EXAMPLE **IFR from Term Structure**

GIVEN

$z_1 = 1\%$, $z_2 = 2\%$, $z_5 = 3\%$ (all annualized spot zero rates).

1-YEAR FORWARD RATE 1 YEAR OUT

$IFR_{1,1} = (1.02)^2 / 1.01 - 1 = \mathbf{3.01\%}$

3-YEAR FORWARD RATE 2 YEARS OUT

$IFR_{2,3} = [(1+z_5)^5 / (1+z_2)^2]^{1/3} - 1 = \mathbf{5.03\%}$ (curriculum example)

LOS B · SHORT RATES

Forward Rates on Market Reference Rates

For money-market rates quoted annually with day-count, the single-period IFR is derived from two adjacent zero-rate deposits:

$$[1 + Z(90)/4] \times [1 + IFR(60)/6] = [1 + Z(150)/2.4]$$

Day counts: 90 days → 12/3 = 4 periods per year; 60 days → 6; 150 days → 2.4.

WORKED EXAMPLE

$IFR_{3m,2m}$ from a MRR Curve — MM Page 7

GIVEN (ANNUALIZED)

Z(30)=1.204%, Z(60)=1.390%, Z(90)=1.568%, Z(120)=1.691%, Z(150)=1.876%.

IFR(60), 90 DAYS FORWARD

IFR(60) = {[1 + 0.01876/2.4] / [1 + 0.01568/4] − 1} × 6 = **2.3289%**

CHECK — CHAIN BACK TO Z(150)

[(1+0.01568/4)·(1+0.023289/6) − 1] × 2.4 = **1.876%** ✓

LOS B · FORWARD RATE AGREEMENTS

FRAs — Advanced Set, Advanced Settled

An **FRA** is a forward on an interest rate. Notation **A×B**: settles at time A, references a (B−A)-period rate. The *long* pays the fixed IFR and receives floating MRR.

- **Advanced set:** the reference rate is observed at time A (the settlement date).
- **Advanced settled:** settlement cash flow is paid at time A, discounted back from B.

Long FRA cash flow (advanced settled)

$$Net_A = PV_A NA \times (MRR_{B-A} - IFR_{A,B-A}) \times Period/360$$

- MRR > IFR → long benefits (rate rose above locked-in rate).
- MRR < IFR → short benefits.

WORKED EXAMPLE **Long 3×6 FRA on $100M — MM Page 9**

GIVEN

NA = $100M. Fixed IFR = 2.15%. Reference MRR at settlement = 2.24299% (90-day). Period = 90/360.

PAY / RECEIVE (SIMPLE INTEREST)

Pay fixed: 100M × 0.0215 × 90/360 = 537,500

Receive fl.: 100M × 0.0224299 × 90/360 = 560,747.50

NET, DISCOUNTED TO SETTLEMENT

Gross net = 560,747.50 − 537,500 = 23,247.50; discounted by (1 + 0.0224299 × 90/360) → advanced-settled net. MRR used for discounting is the fixing, not the IFR.

LOS B · USES OF FORWARD RATES

Uses of Forward Rates & Implied Forwards

Why forward rates matter

- **Lock-in future funding cost** — an issuer going to roll 3-month paper in 6 months can fix the rate today with a long FRA.
- **Benchmarking** — the IFR is the breakeven between investing long-maturity vs rolling short-maturity zeros.
- **Pricing swaps** — a fixed swap rate equates PV of floating payments (set from IFRs) to PV of fixed (the par swap rate).
- **Arbitrage check** — any deviation between the observed FRA rate and the no-arb IFR creates a riskless trade.

Breakeven intuition: Investing at z_B for B periods must equal investing at z_A for A periods then rolling at $IFR_{A,B-A}$ for B−A periods — otherwise arbitrage.

QUIZ 2 · FORWARD RATES & FRAS

Rate Math Drills

Q6. Given z_1=1%, z_2=2%, z_3=3% (all annually compounded), $IFR_{1,1}$ equals:

Q7. Using the same curve, $IFR_{2,1}$ (1-year rate, 2 years forward) equals:

Q8. A long FRA locks in a future fixed rate. At settlement, if the MRR is *above* the FRA rate, the long position:

Q9. An FRA is said to be *advanced set, advanced settled*, meaning:

Q10. The forward-rate model $(1+z_A)^A(1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$ ensures:

QUIZ 3 · INTEGRATION

Putting It All Together

Q11. In the 60-stock example with income, cost, and a 9-month forward, $F_0(T)$ is closest to:

Q12. At t=6m in the same problem with S_t=61.60, $V_t(T)$ is closest to:

Q13. A widening $r_P - r_B$ most directly supports:

Q14. In the ZAR/EUR short 6-mo example, after 2 months with S_t=17.02 and rates (3%, −0.3%), $V_t(T)$ per EUR is closest to:

Q15. Which statement best contrasts an FRA and a swap?

MODULE COMPLETE

Your Scorecard

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LM05 Key Takeaways

Price vs Value

1. Forward **price** $F_0(T)$ is locked at initiation; **value** $V_0(T) = 0$ and $V_t(T)$ changes over time.
2. $V_t(T) = S_t - F_0(T)/(1+r)^{T-t}$; with cash flows adjust S_t for PV of remaining income/costs.
3. At T: $V_T(T) = S_T - F_0(T)$ for the long.

Numeric Anchors

4. 50-stock 6m forward: $F_0(T)$=50.50. With r=1.6% + 2m passage + S_t=50.25 → V_t=0.02.
5. 60-stock with I (0.50 twice) & c (0.25 twice) → $F_0(T)$=60.57. V_t at 6m with S_t=61.60 → 1.18.
6. Long USD/EUR at 1.2010: V_t=0.00295 USD/EUR → 147,500 USD on €50M.

Forward Rates & FRAs

7. IFR identity: $(1+z_A)^A(1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$.
8. With z_1=1%, z_2=2%, z_3=3%: $IFR_{1,1}$=3.01%, $IFR_{2,1}$=5.03%.
9. FRA = single, advanced-set advanced-settled cash flow. Long pays fixed, receives floating.

Last-Minute Checklist

$\mathbf{V_0 = 0 \cdot V_T = S_T - F_0(T) \cdot V_t = S_t - F_0(T)/(1+r)^{T-t}}$

FX: $V_t = S_t - F_0(T) \cdot e^{-(r_P - r_B)(T-t)}$

IFR: $(1+z_A)^A (1+IFR)^{B-A} = (1+z_B)^B$

Long FRA wins if MRR > IFR

Step 1 of 13