LM06 · DERIVATIVES · CFA LEVEL I

Pricing & Valuation of Futures Contracts

Daily MTM, margining, and why forward ≠ futures price

What This Reading Covers

A **futures** contract is an exchange-traded forward — same no-arb pricing $f_0(T) = S_0(1+r)^T$, but **daily mark-to-market** resets $V_t(T)$ to zero after each settlement. Forward and futures prices are equal only when rates are constant or uncorrelated with the futures price.

Learning Outcome Statements

LOS a Compare the value and price of forward and futures contracts

LOS b Explain why forward and futures prices differ

Exam Tip: Big idea: cumulative P&L is ~identical; the timing of cash flows differs. Correlation between futures prices and interest rates is what drives the pricing wedge.

LOS A · MECHANICS

How Futures Work — Margin, MTM, Reset

Notation	$F_t(T)$ = forward price; $f_t(T)$ = futures price
At initiation	$f_0(T) = S_0(1+r)^T$; $V_0(T) = 0$; post initial margin
Daily	$V_t(T)$ reset to 0 after MTM. $\Delta f_t(T)$ = daily MTM cash flow, not $V_t(T)$.
Maintenance margin	Falling below triggers a margin call; must top up to initial level
At T	$f_T(T) = S_T$; cumulative P&L = $S_T - f_0(T)$ (same as forward in total)

Daily settlement resets the contract to zero value each day. You pay gains/losses in cash; the quoted $f_t(T)$ floats with the market.

LOS A · WORKED EXAMPLE

Gold Futures — MTM vs Forward Value

WORKED EXAMPLE **Gold, S_0=1770, 100-oz Contract — MM Page 1**

GIVEN

S_0=1,770; r=2%; T=91 days. Initial margin = $4,950; maintenance =$4,450. Long position.

INITIATION

$F_0(T) = f_0(T) = 1{,}770 \cdot (1.02)^{91/365}$ = **1,778.76**. $V_0(T) = 0$ for both.

DAY 1: $F_1(T)$ = 1,773.76 (↓ $5)

Futures: 100 oz × $5 = **$500 MTM loss**; $V_1(T)$ reset to 0.

$S_1 = 1{,}773.76 / (1.02)^{90/365} = 1{,}765.12$.

Forward value: $V_1(T) = 1{,}765.12 - 1{,}778.76/(1.02)^{90/365}$ = **−4.98** (× 100 oz = −498).

DAY 2: $F_2(T)$ = 1,769.76 (↓ $4)

Futures: $400 additional MTM loss; cumulative −$900 cash; $V_2(T)$ reset to 0.

Forward: $V_2(T) = 1{,}761.24 - 1{,}778.76/(1.02)^{89/365}$ = **−8.95** (× 100 oz = −895).

Exam Tip: Forward: $V_t(T)$ builds up cumulative gain/loss. Futures: $V_t(T)$ is *always 0* right after MTM — cash flows happen daily instead.

LOS A · COMPARISON

Side-by-Side Comparison

DIMENSION	FORWARD	FUTURES
Price	$F_0(T)$ fixed	$f_t(T)$ changes daily
Value	$V_t(T) = S_t - PV[F_0(T)]$	$V_t(T) = 0$ at end of each day
Cash flows	At maturity only	Daily (MTM)
Margin	None (usually)	Initial + maintenance
Counterparty	OTC — both sides	Exchange / CCP
Cumulative P&L	$S_T - F_0(T)$	$S_T - f_0(T)$ (identical in total if prices = forwards)

Gains/losses at T are *identical* in magnitude; only the *timing* differs.

LOS A · MRR FUTURES

Interest Rate Futures — Index Basis

$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})\ (decimal)$$
$$f_{A,B-A} = 100 - MRR_{A,B-A}\ (percent)$$

Long position (*long index = long bond*) earns the fixed rate. Short position pays the fixed rate.

WORKED EXAMPLE **MRR Futures — MM Page 2**

GIVEN

$f_{A,B+A}$ = 98.25 → $MRR_{A,B+A}$ = 1.75%. Long position pays receives the fixed 1.75%.

SCENARIO: MRR ENDS AT 1%

Index settles at 99; contract pays **0.75%** × NA to long.

SHORT SCENARIO: MRR ENDS AT 2%

Index settles at 98; contract pays 0.25% × NA to short. Short is hedged: borrow at MRR=2%, offset 0.25% from contract gain.

QUIZ 1 · MECHANICS & VALUES

Futures vs Forward Mechanics

Q1. Immediately after daily settlement, the value of a futures contract is:

Q2. At t=91 days, S_0=1,770 and r=2%. $f_0(T)$ is closest to:

Q3. In the gold example, $f_1(T)$ drops $5 on Day 1 on a long 100-oz contract. The MTM cash flow is:

Q4. In the same example, the Day-1 forward *value* per oz is closest to:

Q5. An interest rate futures index of 98.25 implies an MRR of:

LOS B · CORRELATION & CONVEXITY

Why Forward ≠ Futures Price in General

Equality conditions

$F_0(T) = f_0(T)$ if **either**:

1. Interest rates are constant, or
2. Futures prices and interest rates are uncorrelated.

Correlation logic

- **Positive correlation (f and r):** long futures reinvest gains at higher r, borrow losses at lower r → *long futures is more desirable than long forward* → futures price > forward price.
- **Negative correlation:** long forward is more desirable → forward price > futures price.

Exam Tip: "More desirable" contract commands a higher price. Magnitude depends on interest rate volatility and the correlation.

LOS A · HEDGING

Hedging Rate Moves — Futures vs FRA

EXPOSURE	HEDGE — FUTURES	HEDGE — FRA
MRR ↑ (borrower)	Short interest-rate futures	Long FRA (pay fixed, receive floating)
MRR ↓ (lender)	Long interest-rate futures	Short FRA

WORKED EXAMPLE **Corporate Borrower, $50M in 6 mos**

GIVEN

Need to borrow $50M for 3 months starting in 6 mos; pay MRR at that time.

SHORT INTEREST RATE FUTURES

If MRR rises, futures price falls → futures gain offsets higher borrowing cost. Long FRA would have the same hedge.

LOS B · BPV & CONVEXITY

Convexity Bias — FRA vs Futures

Basis point value (BPV) per futures contract

$$BPV = NA \times 0.0001 \times Period/360$$

Example: 1-month, 3-month 90-day contract, NA = $1,000,000:

BPV = 1,000,000 × 0.0001 × 90/360 = **$25 per bp**.

WORKED EXAMPLE **FRA vs Futures — One bp Move**

GIVEN

Short 3-mo FRA at 2.21%; rate moves +1 bp to 2.22%. NA = $1M.

FRA NET PAYMENT (DISCOUNTED, 90-DAY)

(0.0222 − 0.0221) × 1M × 90/360 / (1 + 0.0222 × 90/360) = **$24.86**

FUTURES MTM

+$25 (linear BPV — no discounting). This linear-vs-discounted gap is the **convexity bias** — futures price > FRA-implied forward rate.

The FRA *discounts* the settlement; the futures *does not*. This asymmetry is the convexity bias.

QUIZ 2 · DIFFERENCES

Equality Conditions & Bias

Q6. Forward and futures prices are *identical* if:

Q7. If futures prices are positively correlated with interest rates, then:

Q8. The BPV of a 90-day interest rate future on $1M notional is:

Q9. A borrower worried about higher MRR should:

Q10. The convexity bias arises because:

LOS B · MARKET STRUCTURE

Central Clearing & Cash Flow Convergence

With **central clearing** of forwards via CCPs (variation margin / collateral), forwards increasingly resemble futures in cash-flow profile. Historically:

- **Futures**: lack of cash for daily MTM → forced liquidation.
- **Bilateral forwards**: no margin; contract survives to expiration through solvent counterparty.
- **Cleared forwards**: initial + variation margin narrow the cash-flow gap vs futures.

> Central clearing narrows — but doesn't fully eliminate — the cash-flow differences driving the forward/futures price wedge.

QUIZ 3 · INTEGRATION

Futures Concepts Integration

Q11. Which is *false* about futures vs forwards at maturity?

Q12. In the gold Day-2 example, the forward value per oz is closest to:

Q13. Long futures gain when:

Q14. The margin structure of futures means:

Q15. Per MM, if MRR moves +1 bp on a 3-mo, $1MFRAat2.21$24.86 while the futures BPV is ~$25. The difference reflects:

MODULE COMPLETE

Your Scorecard

0 / 15

LM06 Key Takeaways

Futures vs Forwards

1. Same no-arb price at inception: $f_0(T) = S_0(1+r)^T$.
2. Forward: $V_t(T)$ accumulates; Futures: $V_t(T)$ reset to 0 daily via MTM.
3. Cumulative P&L at T is approximately the same — timing of cash flows differs.

Equality Conditions & Bias

4. F = f when rates are constant or futures and rates are uncorrelated.
5. Positive correlation (f,r) → long futures more desirable → f > F.
6. Convexity bias: FRA discounts settlement, futures linear BPV ($24.86vs$25 per bp on $1M/90d).

Numeric Anchors

7. Gold: S_0=1,770, r=2%, T=91d → $f_0(T)$=1,778.76. Day 1 ↓$5 \to -$500 MTM; V_1(forward) = −4.98/oz (−498).
8. Day 2 forward value −8.95/oz (−895 total).
9. Rate future index 98.25 → MRR 1.75%. Long earns fixed 1.75%.

Last-Minute Checklist

$f_0(T) = S_0(1+r)^T$ · V_t(futures) = 0 after MTM · $f_t(T)$ = 100 − MRR

Hedge MRR↑ = short futures / long FRA · Hedge MRR↓ = long futures / short FRA

BPV = NA × 0.0001 × period/360 · Convexity bias from FRA discounting

Central clearing narrows cash-flow gap

Step 1 of 13