LM07 · DERIVATIVES · CFA LEVEL I

Pricing & Valuation of Interest Rate and Other Swaps

Swap = a series of FRAs with one common fixed rate — the par swap rate

What This Reading Covers

A swap is an agreement to exchange a *series* of cash flows; an FRA settles a *single* cash flow. At initiation V_0(swap) = 0 via choice of the **fixed rate r_{fx}**, which equates PV(floating payments) = PV(fixed payments). The swap can be replicated as a portfolio of FRAs at different rates.

Learning Outcome Statements

LOS a Describe how swap contracts are similar to but different from a series of forward contracts

LOS b Contrast the value and price of swaps

Exam Tip: Par-swap-rate formula: $r_{fx} = (1 - DF_n) / \Sigma\, DF$ — lock in; always compute discount factors $DF_i = 1/(1+z_i)^i$ first.

LOS A · REVIEW

Building Blocks — Zeros, DFs, IFRs

From spot zeros to IFRs

With z_1=1%, z_2=2%, z_3=3%:

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IFR_{1,1} = (1.02)^{2}/1.01 - 1 = \mathbf{3.01%} \\ IFR_{2,1} =
(1.03)^{3}/(1.02)^{2} - 1 = \mathbf{5.03%} \\ Check: 1.01 \;
\times \; 1.0301 \; \times \; 1.0503 = 1.0927 = (1.03)^{3} ✓
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Discount factors

$$DF_1 = 1/1.01 = \mathbf{0.990099}$$
$$DF_2 = 1/(1.02)^2 = \mathbf{0.961169}$$
$$DF_3 = 1/(1.03)^3 = \mathbf{0.915142}$$
$$\Sigma DF = \mathbf{2.86640}$$

> The floating leg of a swap on a $1 notional is equivalent to a *par floating-rate bond* — it prices to par when coupons are MRRs discounted at the zero curve.

LOS B · FIXED RATE DERIVATION

Deriving the Par Swap Rate r_{fx}

From fixed-bond parity

$$1 = \text{PMT} \times (DF_1 + DF_2 + DF_3) + DF_3$$
$$\Rightarrow 1 - DF_n = \text{PMT} \times \Sigma DF$$
$$\Rightarrow \text{PMT} = \mathbf{(1 - DF_n)/\Sigma DF} = r_{fx}$$

WORKED EXAMPLE **3-Period Swap — MM Page 3**

GIVEN

z_1=1%, z_2=2%, z_3=3%. Notional $1M. Fixed vs floating MRR annually.

DISCOUNT FACTORS

DF_1=0.990099; DF_2=0.961169; DF_3=0.915142. Σ=2.86640.

PAR SWAP RATE

r_{fx} = (1 − 0.915142) / 2.86640 = **2.9604%**

Exam Tip: The par swap rate is the *multi-period breakeven*: an investor is indifferent between paying r_{fx} for n periods or paying the IFRs for each sub-period.

LOS B · V_0 = 0

Verifying V_0(swap) = 0

Setup on $1M notional, fixed 2.9604%

Floating payments at end of each period = MRR for that period × $1M.

PERIOD	FLOATING (MRR)	CF	PV FLOATING	FIXED (2.9604%)	PV FIXED
t=1	1%	+10,000	9,900.99	−29,604	−29,310.89
t=2	3.01% ($IFR_{1,1}$)	+30,099	28,930.22	−29,604	−28,454.44
t=3	5.03% ($IFR_{2,1}$)	+50,295	46,027.05	−29,604	−27,091.85
Sum	—	—	**84,858.26**	—	**−84,857.18**

PV(fl.) ≈ PV(fx.) → **V_0(swap) ≈ 0** (small rounding discrepancy).

QUIZ 1 · PRICING THE SWAP

Fixed Rate & Pricing Drills

Q1. The par swap rate r_{fx} is given by:

Q2. With z_1=1%, z_2=2%, z_3=3%, the 3-period par swap rate is closest to:

Q3. The value of a plain-vanilla interest rate swap at initiation is:

Q4. In the 3-period example on $1M with z_1=1%, z_2=2%, z_3=3%, PV of the floating leg (full term) is closest to:

Q5. The floating leg of an interest rate swap is most like:

LOS A · DECOMPOSITION

Swap as a Series of FRAs

Key contrast

- A **3-period swap** has *one* fixed rate r_{fx} applied to each settlement.
- A **portfolio of 3 FRAs** has *three different* fixed rates (the IFRs: 1%, 3.01%, 5.03%).
- Both have the same total PV at initiation (both zero-value) but different period-by-period cash flows.

Swap = multiple settlements at end-of-period, one fixed rate.

FRAs = single settlement per contract at start-of-period, advanced-settled, different fixed rates.

EQUIVALENCE **Investor Indifference**

EQUIVALENT PORTFOLIOS

Deposit @ 1% for 1 yr + short $FRA_{1,1}$ @ 3.01% = deposit @ 2% for 2 yrs. Extending: + short $FRA_{2,1}$ @ 5.03% = deposit @ 3% for 3 yrs.

SWAP EQUIVALENCE

Borrow @ 1% + long 3-period swap at 2.9604% ≈ borrow @ 3% for 3 years.

LOS B · NET PAYMENTS

Periodic Net Settlement (S_N)

$$S_N = (MRR - r_{fx}) \times NA \times Period$$

Example: (0.01 − 0.029604) × 1M × 1 = **−19,604** — the fixed payer pays 19,604 at t=1 (net of floating receipt).

Observation timing

- $MRR_{0,1}$ is *known* at inception (just the current 1-year rate).
- $MRR_{1,1}$ and $MRR_{2,1}$ are only known at t=1 and t=2 respectively.
- As time passes, V_t(swap) ≠ 0 because realized vs expected floating rates differ.

LOS B · MID-LIFE VALUE

Value of a Swap During Its Life

WORKED EXAMPLE **Receive-Floating Swap at Each Period — MM Page 5**

SETUP

\$1M NA, r_{fx}=2.9604%, floating CFs: +10,000 / +30,099 / +50,295. Fixed CFs: −29,604 each.

AT T=1 (JUST AFTER FIRST SETTLEMENT)

Remaining floating: +30,099 / +50,295 → PV(fl.) = 74,957.27. Remaining fixed: −29,604 / −29,604 → PV(fx.) = 55,546.29.

V_1(swap) = 74,957.27 − 55,546.29 = **19,410.98**

AT T=2

Remaining floating: +50,295 → PV = 46,027.05; remaining fixed: −29,604 → PV = 27,091.85.

V_2(swap) = **18,935.20**

AT T=3

Last CFs net to: 50,295 − 29,604 = 20,691 — paid; then V_3(swap) = **0**.

V_t(swap) changes as (1) time passes, (2) the zero/forward curve shifts — the receiver-floating party benefits when expected floating rates rise above the locked-in fixed.

QUIZ 2 · MID-LIFE VALUES

Life & Periodic Settlements

Q6. The periodic settlement value (S_N) for a fixed-payer receive-floating swap is:

Q7. In the 3-period swap example at t=1, V_1(swap) to the receive-fl. side is closest to:

Q8. Which rate is known at swap inception?

Q9. An increase in expected future floating rates will:

Q10. A 3-period swap and 3 separate FRAs at the IFRs:

LOS A · CURRENCY & EQUITY SWAPS

Currency & Equity Swaps — Brief Survey

Currency swap

Parties exchange principals in two currencies at inception and reverse at maturity. Interest payments are made in each currency using its fixed or floating rate. Used for cross-border funding.

Equity swap

Exchange of the return on an equity index/portfolio for an interest payment (fixed or floating) on a debt instrument. Used to gain/hedge equity exposure synthetically.

All swaps share: (1) $V_0 = 0$ at initiation, (2) V_t drifts with realized vs expected cash flows, (3) counterparty credit risk is present.

LOS A · RECAP

FRA vs Swap — Quick Recap

DIMENSION	FRA	SWAP
Settlements	Single, at start of period	Multiple, at end of each period
Fixed rate	One IFR per contract	One par rate for all periods
Initial value	0	0
Payoff profile	Symmetric (MRR − IFR)	Symmetric (MRR − r_{fx})
Counterparty risk	Yes	Yes (grows as swap life passes)
Settlement math	Advanced set, advanced settled (discounted)	Period-end, typically undiscounted

QUIZ 3 · INTEGRATION

Swap Integration

Q11. At t=2 in the 3-period example, V_2(swap) to the receiver-fl. side is closest to:

Q12. The net fixed-payer settlement at t=1 with MRR=1% and r_{fx}=2.9604% on $1M is:

Q13. Which is *true* about the par swap rate?

Q14. Compared to a single FRA, counterparty risk of a 5-year swap:

Q15. A currency swap differs from an interest rate swap primarily because:

MODULE COMPLETE

Your Scorecard

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LM07 Key Takeaways

Swap Pricing

1. Par swap rate: $r_{fx} = (1 - DF_n) / \Sigma$ DF.
2. V_0(swap) = 0 — chosen by construction.
3. A swap = series of FRAs with one common fixed rate vs different IFRs per period.

Numeric Anchors

4. z_1=1%, z_2=2%, z_3=3% → $IFR_{1,1}$=3.01%, $IFR_{2,1}$=5.03%; r_{fx}=2.9604%.
5. On $1M: V_1(swap)=19,411; V_2(swap)=18,935; V_3(swap)=0.
6. Periodic net: $S_N = (MRR - r_{fx}) \times NA \times$ Period. t=1 net = −19,604 for fx. payer.

FRA vs Swap

7. FRA: single advanced-settled payment. Swap: multiple end-of-period settlements.
8. Rising expected floating rates → positive value for fixed payer.
9. Counterparty risk grows with swap life; currency swaps also exchange principals.

Last-Minute Checklist

r_{fx} = (1 − DF_n) / Σ DF · V_0 = 0 · PV(fl.) = PV(fx.) at inception

Floating leg = par floating bond · Swap ≈ series of FRAs at different IFRs

S_N = (MRR − r_{fx}) × NA × Period

Fixed payer gains when expected floating rates rise

Step 1 of 13