LM08 · DERIVATIVES · CFA LEVEL I

Pricing & Valuation of Options

Exercise value, moneyness, time value, arbitrage bounds, replication, and the six pricing factors

What This Reading Covers

Options are **contingent claims** — their payoff depends on whether the underlying crosses the exercise price. All options here are *European* on non-dividend-paying stock. We split option value into *exercise value* + *time value*, bound the price by arbitrage, build a replication portfolio (via delta), and enumerate six factors that move C_t and P_t.

Learning Outcome Statements

LOS a Explain the exercise value, moneyness, and time value of an option

LOS b Contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims

LOS c Identify the factors that determine the value of an option and describe how each factor affects the value of an option

Exam Tip: Option premium = **exercise value + time value**. TV ≥ 0 and decays to 0 at expiration.

LOS A · EXERCISE VALUE

Exercise Value — At Expiration and Any Time t

At expiration (T)

$$Call_T = max(0, S_T - X) \quad Put_T = max(0, X - S_T)$$

If the payoff is negative, the option expires worthless — the holder loses only the premium paid.

At any time t < T

Exercise value discounts the strike to time t so amounts are compared at the same point in time:

$$Callexercisevalue = max(0, S_t - X/(1+r)^{T-t})$$
$$Putexercisevalue = max(0, X/(1+r)^{T-t} - S_t)$$

Key Insight

Because discounting shrinks X, the call's exercise value grows with r (and T), while the put's shrinks — this is the seed of the "r up → C up, P down" rule later.

LOS A · MONEYNESS

Moneyness — ITM, ATM, OTM

	CALL OPTION	PUT OPTION
In-the-money (ITM)	$S_t > X$	$S_t < X$
At-the-money (ATM)	$S_t = X$	$S_t = X$
Out-of-the-money (OTM)	$S_t < X$	$S_t > X$

Price ladder intuition

Rising S_t walks a call from OTM → ATM → ITM (value rises). The same move walks a put from ITM → ATM → OTM (value falls). Moneyness describes where you stand today relative to the strike.

Exam Tip: Moneyness is about the *spot* relative to strike — it is NOT the same as the option being profitable. An OTM option with a tiny premium can still turn a profit if S moves enough before T.

LOS A · TIME VALUE

Time Value — Always Positive, Decays to Zero

Prior to expiration

$$C_t = max(0, S_t - X/(1+r)^{T-t}) + TV$$
$$P_t = max(0, X/(1+r)^{T-t} - S_t) + TV$$

Time value (TV) is the extra premium above exercise value — the value of *optionality* between now and T.

Behavior of TV

- TV > 0 prior to expiration for every option with time left
- TV = 0 at expiration (C_T and P_T collapse to intrinsic value)
- TV **decays very fast in the last few weeks** (theta burn)
- At-the-money options carry the most TV because either direction is equally "useful"

Option premium decomposition

Premium = Exercise Value + Time Value. OTM options have zero exercise value — the entire premium is TV.

QUIZ 1 · EXERCISE VALUE, MONEYNESS, TIME VALUE

Check Your Understanding — Quiz 1

Q1. A European call with X = $50tradesat$6.20 when S_t = $52, T−t = 0.5 yr, r = 4%. What is its time value?

Q2. Stock trades at $40;putstrikeis$45. The put is:

Q3. As expiration approaches, the time value of an option:

Q4. An OTM call has premium = $0.80. Its exercise value and TV are:

Q5. A call's exercise value at time t is max(0, S_t − X/$(1+r)^{T-t}$). Why is X discounted?

LOS B · PAYOFF & ARBITRAGE

Asymmetric Payoffs — Contingent Claims vs. Forwards

Profit equations

$$\Pi_{Call} = max(0, S_T - X) - C_0$$
$$\Pi_{Put} = max(0, X - S_T) - P_0$$

Forwards have a *symmetric* payoff (gains mirror losses around F_0). Options have an *asymmetric* payoff: bounded loss (premium) and, for calls, unbounded upside.

Forward vs Option — arbitrage vs replication

Forwards are priced by a **pure no-arbitrage argument**: $F_0 = S_0(1+r)^T$. Options are *bounded* by arbitrage but their exact value requires **replication** — constructing a portfolio that matches the option's payoff.

LOS B · NO-ARBITRAGE BOUNDS

No-Arbitrage Upper & Lower Bounds

Call bounds

$$max(0, S_t - X/(1+r)^{T-t}) < C_t \ \leq \ S_t$$

Put bounds

$$max(0, X/(1+r)^{T-t} - S_t) < P_t \ \leq \ X$$

Why these bounds?

- **Call upper bound = S_t:** the right to buy at X can never be worth more than owning the stock outright.
- **Put upper bound = X:** the right to sell at X tops out when the stock goes to zero.
- **Lower bound = exercise value:** option must be worth at least intrinsic value (else buy & exercise for free profit).

Exam Tip: If C_t or P_t is quoted *outside* these ranges, arbitrage is available — a certain guaranteed exam trap.

LOS B · REPLICATION

Replication via Delta

Replicating a call (Δ = 0.40 example)

- Borrow $0.40 \times X/(1+r)^{T-t}$ at the risk-free rate
- Buy 0.40 shares of the underlying

As Δ changes (with S, time, vol), the replicating portfolio is re-balanced.

Replicating a put (Δ = 0.40 example)

- Short 0.40 shares of the underlying
- Lend $0.40 \times X/(1+r)^{T-t}$ at the risk-free rate

Key idea

A call behaves like a *leveraged long position* in the stock (borrow + buy Δ shares). A put behaves like a *cash-collateralized short* (lend + sell Δ shares). Δ quantifies how many shares are needed per option contract.

LOS C · FACTOR 1 — S_T

Factor 1 — Value of the Underlying (S)

Call

Higher S_t → higher C_t and higher Δ

ITM calls move ~1:1 with the stock (Δ → 1)

Put

Higher S_t → lower P_t and less-negative Δ

ITM puts move ~−1:1 (Δ → −1)

Delta intuition

The magnitude of option-value change for a small change in S is approximated by its delta Δ. Calls have $\Delta \in (0, 1)$; puts have $\Delta \in (-1, 0)$.

QUIZ 2 · ARBITRAGE, REPLICATION, UNDERLYING FACTOR

Check Your Understanding — Quiz 2

Q6. A call option quotes C_t = $32 when S_t = $30. What arbitrage is available?

Q7. To replicate a call with Δ = 0.25 you should:

Q8. Forwards use pure no-arbitrage pricing, while options require:

Q9. Put option lower bound at time t is:

Q10. Rising S affects Δ how?

LOS C · FACTORS 2 & 3

Exercise Price (X) & Time to Expiration (T)

Factor 2 — Exercise price X

HIGHER X	CALL VALUE	PUT VALUE
Effect	Decreases	Increases
Intuition	Harder to reach X	Easier to sell above X

Factor 3 — Time to expiration T

For European options on non-dividend stocks:

- **Call:** longer T → higher C (more chance to finish ITM; X is discounted more deeply)

- **Put:** generally longer T → higher P, BUT can be ambiguous (longer T = less-PV(X) lowers put too)

Visualizing T

Plot option value vs. S for 3, 2, and 1 months to expiration. The curves stack top-to-bottom for calls (3 mo highest); for puts they usually stack the same way but the ordering can invert deep ITM when r is large.

LOS C · FACTORS 4, 5, 6

Rate, Volatility, Income/Costs

Factor 4 — Risk-free rate r

$$Call : max(0, S_t - X/(1+r)^{T-t}) \uparrow when r \uparrow$$
$$Put : max(0, X/(1+r)^{T-t} - S_t) \downarrow when r \uparrow$$

Higher r discounts X more → call value ↑, put value ↓.

Factor 5 — Volatility σ

Higher volatility raises *both* call and put premiums. The payoff is asymmetric: more vol boosts upside potential without hurting downside (which is capped at the premium).

Factor 6 — Income / Costs on the underlying

	CALL	PUT
Benefits (dividends, coupons)	Decrease	Increase
Costs (storage, insurance)	Increase	Decrease

Benefits lower the forward price, so the call on a lower F is worth less; the put is worth more. Costs do the opposite.

LOS C · SUMMARY

All Six Factors — Master Table

INCREASE IN	CALL VALUE	PUT VALUE
1. Value of the underlying (S)	Increase	Decrease
2. Exercise price (X)	Decrease	Increase
3. Time to expiration (T)	Increase	Increase or Decrease
4. Risk-free rate (r)	Increase	Decrease
5. Volatility (σ)	Increase	Increase
6. Benefits (dividends/income)	Decrease	Increase
6. Costs (storage/insurance)	Increase	Decrease

Quick mnemonic

Calls love: higher S, lower X, more T, higher r, higher σ, higher costs, lower benefits.

Puts love: lower S, higher X, more T (usually), lower r, higher σ, lower costs, higher benefits.

QUIZ 3 · FACTOR EFFECTS

Check Your Understanding — Quiz 3

Q11. All else equal, which increases BOTH call and put values?

Q12. Rising r has what effect?

Q13. A stock pays a large upcoming dividend. This will:

Q14. Which is the correct pairing for the call's exercise-value expression?

Q15. A European call and put share the same S, X, T, r, σ. If costs of holding the underlying rise (e.g., storage), then:

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Your Quiz Score

0 / 15

Key Takeaways

- **Premium = Exercise value + Time value.** TV ≥ 0 and decays to 0 at T.
- Moneyness: call ITM when S > X; put ITM when S < X.
- Arbitrage bounds: max(0, S − PV(X)) < C ≤ S · max(0, PV(X) − S) < P ≤ X.
- Replication: call = borrow Δ·PV(X) + buy Δ shares; put = lend Δ·PV(X) + sell Δ shares.

- Six factors: S, X, T, r, σ, income/costs.

Last-Minute Checklist

1. Call exercise value at t: $\max(0, S_t - X/(1+r)^{T-t})$
2. Put exercise value at t: $\max(0, X/(1+r)^{T-t} - S_t)$
3. $C \le S$, $P \le X$ — upper bounds
4. σ ↑ helps BOTH calls and puts
5. r ↑ → C ↑, P ↓ · Dividends → C ↓, P ↑ · Costs → C ↑, P ↓

Exam-Day Traps

(1) Don't confuse "ITM" with "profitable" — ITM ignores premium paid.

(2) Use PV(X) for exercise value at $t \neq T$.

(3) Remember time value is ALWAYS ≥ 0 and accelerates downward near expiration.

(4) Forwards use arbitrage; options use replication — different pricing mechanisms.

Step 1 of 15