LM09 · DERIVATIVES · CFA LEVEL I

Option Replication Using Put-Call Parity

Fiduciary call = Protective put — the identity that links every European option price

What This Reading Covers

Put-call parity is the foundation equation tying together four prices: the European call C_0, the European put P_0, the spot S_0, and the PV of the exercise price. Two portfolios — a *fiduciary call* (call + ZCB) and a *protective put* (stock + put) — must have identical payoffs at T and therefore identical prices at t=0. Rearranging creates synthetic longs/shorts and extends to the forward version (put-call-forward parity).

Learning Outcome Statements

LOS a Explain put-call parity for European options

LOS b Explain put-call forward parity for European options

Exam Tip: The parity identity: $\mathbf{S_0 + P_0 = C_0 + X/(1+r)^T}$. Given any three of these, solve the fourth.

LOS A · REPLICATING THE UPSIDE

Two Strategies with Identical Payoffs

Goal: capture upside without the downside

Strategy 1 — Fiduciary Call

Long call (strike X) + invest $X/(1+r)^T$ in a zero-coupon bond (grows to X at T)

Cost = $C_0 + X/(1+r)^T$

Strategy 2 — Protective Put

Long stock + long put (strike X)

Cost = $S_0 + P_0$

Payoffs at T — identical

OUTCOME	$S_T < X$	$S_T = X$	$S_T > X$
Protective put	X	$S_T = X$	S_T
Fiduciary call	X	$X = S_T$	S_T

Because both portfolios pay the same at every possible S_T, their **prices must also match** — otherwise an arbitrage opportunity exists.

LOS A · THE IDENTITY

Put-Call Parity — The Identity

$$\mathbf{S_0 + P_0 = C_0 + X/(1+r)^T}$$

Every European option pair on a non-dividend stock obeys this. Given three inputs you can always solve for the fourth.

WORKED EXAMPLE 1

Solve for the put price (IFT)

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European call and put with X = \$100. S_0 = \$90. r = 10%. T = 3 months (0.25 yr). Call price C_0 = \$2.

SOLUTION

$P_0 = C_0 + X/(1+r)^T - S_0$

$P_0 = 2 + 100/1.10^{0.25} - 90$

$P_0 = 2 + 97.64 - 90$

ANSWER

P_0 = \$9.64

LOS A · REARRANGEMENTS

Synthetic Positions via Rearrangement

Solve parity for each input to build a synthetic replica of any single instrument from the other three:

$$\mathbf{S_0 = C_0 - P_0 + X/(1+r)^T} \; synthetic long stock$$

$$\mathbf{P_0 = C_0 - S_0 + X/(1+r)^T} \; synthetic long put$$

$$\mathbf{C_0 = S_0 + P_0 - X/(1+r)^T} \; synthetic long call$$

$$\mathbf{X/(1+r)^T = S_0 + P_0 - C_0} \; synthetic long bond$$

Synthetic shorts

Multiply both sides by −1 to get the short equivalents. Example: $-S_0 = -C_0 + P_0 - X/(1+r)^T$ is a **synthetic short stock** (short call, long put, short bond).

Exam Tip: Whenever a vignette says "construct a synthetic X," set up parity and solve for X.

QUIZ 1 · PUT-CALL PARITY MECHANICS

Check Your Understanding — Quiz 1

Q1. Put-call parity states:

Q2. Using the IFT example (X=100, S_0=90, r=10%, T=0.25, C_0=2), P_0 equals:

Q3. A synthetic long call is:

Q4. Two portfolios with identical time-T payoffs must today have:

Q5. Synthetic long bond equals:

LOS B · PUT-CALL-FORWARD PARITY

Put-Call-Forward Parity

Replacing the stock with a forward

Since $F_0(T) = S_0(1+r)^T$, we have $S_0 = F_0(T)/(1+r)^T$. Substituting into parity:

$$\mathbf{F_0(T)/(1+r)^T + P_0 = C_0 + X/(1+r)^T}$$

Left side = *synthetic protective put* (cash position + put). Right side = *fiduciary call*. The forward is costless to enter — the cash position is $PV(F_0(T))$.

Payoff check — all equal

	$S_T < X$	$S_T = X$	$S_T > X$
Put P_0	$X - S_T$	0	0
Forward $F_0(T)$	$S_T - F_0(T)$	$S_T - F_0(T)$	$S_T - F_0(T)$
Cash $F_0(T)/(1+r)^T$	$F_0(T)$	$F_0(T)$	$F_0(T)$
Total	X	S_T	S_T

LOS B · NUMERIC EXAMPLE

Forward Parity — Numeric Walkthrough

WORKED EXAMPLE 2 **Forward replacing spot (IFT)**

GIVEN

Same setup as Ex. 1: $X=100$, $S_0=90$, $r=10\%$, $T=0.25$. $C_0=2$. Now use a 3-month forward on the stock.

STEP 1 — FORWARD PRICE

$F_0(T) = S_0(1+r)^T = 90 \times 1.10^{0.25} = \mathbf{\$92.17}$

STEP 2 — SOLVE FOR PUT

$P_0 = C_0 + X/(1+r)^T - F_0(T)/(1+r)^T$

$P_0 = 2 + 100/1.10^{0.25} - 92.17/1.10^{0.25}$

$P_0 = 2 + 97.64 - 83.76$

ANSWER

$\mathbf{P_0 = \$9.64}$ — identical to the spot version, confirming forward parity.

LOS B · SYNTHETIC FORWARD POSITIONS

Synthetic Long & Short Forwards

Rearrange forward parity

$$\mathbf{[F_0(T) - X]/(1 + r)^T = C_0 - P_0}$$

Left side is the PV of a long forward payoff (you buy at $F_0(T)$ and receive something worth X at strike). Right side says a long call + short put replicates a long forward.

Synthetic short forward

$$\mathbf{[X - F_0(T)]/(1 + r)^T = P_0 - C_0}$$

Long put + short call = short forward position.

Two summary equations

$$F_0(T)/(1 + r)^T = C_0 - P_0 + X/(1 + r)^T \; synthetic long forward$$

$$-F_0(T)/(1 + r)^T = P_0 - C_0 - X/(1 + r)^T \; synthetic short forward$$

QUIZ 2 · FORWARD PARITY & SYNTHETIC FORWARDS

Check Your Understanding — Quiz 2

Q6. Put-call-forward parity replaces S_0 with:

Q7. With S_0 = \$90 and r = 10% for 3 months, $F_0(T)$ is:

Q8. A synthetic long forward = :

Q9. The fiduciary call costs:

Q10. Using forward parity with P_0 from Example 2 ($F_0(T)$=92.17, X=100, r=10%, T=0.25, C_0=2):

LOS B · APPLICATION — CREDIT ANALYSIS

Parity Applied to Credit Risk

Firm value decomposition

Consider a firm with equity E and zero-coupon debt with face value D due at T:

$$\mathbf{V_0} = \mathbf{E_0} + \mathrm{PV(D)}$$

AT TIME T	DEBT HOLDERS	SHAREHOLDERS
$V_T > D$	Get D	Get $V_T - D$
$V_T \leq D$	Get V_T	Get 0

Positions in option terms

- **Debt holders**: limited upside (max D), unlimited downside → $\min(V_T, D)$ = long bond + *short put* with strike D
- **Shareholders**: limited downside (floor 0), unlimited upside → $\max(0, V_T - D)$ = *long call* with strike D

LOS B · VALUING RISKY DEBT

$V_0 = C_0 + PV(D) - P_0$

Deriving the credit-risk version

Starting from parity with X = D and S → V:

$$V_0 + P_0 = C_0 + \mathrm{D}/(1+r)^T$$

$$\mathbf{V_0 = C_0 + [D/(1+r)^T - P_0]}$$

Interpretation: firm value = equity (long call) + **risky debt** (risk-free debt minus a short put).

Credit risk embedded in P_0

The value of the put (P_0) equals the expected cost of default. **Pricing the put therefore quantifies the credit risk** of the debt — and can be interpreted as the credit spread.

Risky debt = Risk-free debt PV(D) − Put(struck at D)

QUIZ 3 · SYNTHETIC POSITIONS & CREDIT RISK

Check Your Understanding — Quiz 3

Q11. In the credit-risk application, equity holders are long a:

Q12. Risky debt can be replicated as:

Q13. A synthetic long stock using options is:

Q14. The put-call-forward parity equation is:

Q15. A violation of put-call parity implies:

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Your Quiz Score

0 / 15

Key Takeaways

- **Put-call parity**: $S_0 + P_0 = C_0 + X/(1+r)^T$
- Fiduciary call (C + PV(X)) and protective put (S + P) have identical payoffs → identical prices
- **Put-call-forward parity**: $F_0(T)/(1+r)^T + P_0 = C_0 + X/(1+r)^T$
- Synthetic long forward = long call − long put; synthetic short forward = long put − long call

- Credit risk: $V_0 = C_0 + PV(D) - P_0$; debt holders are short a put on firm value

Last-Minute Checklist

1. Memorize parity and the four rearrangements for synthetic long/short positions.
2. Example: $X=100$, $S_0=90$, $r=10\%$, $T=0.25$, $C_0=2 \rightarrow P_0 = \$9.64$.
3. Forward version: $F_0(T) = 90 \times 1.10^{0.25} = \$92.17$; same $P_0 = \$9.64$.
4. Equity = long call on V (X=D); Risky debt = $PV(D) - P_0$.
5. Any parity violation = arbitrage.

Step 1 of 13