

LM08 · ECONOMICS · CFA LEVEL I

Exchange Rate Calculations

Cross rates, forward rates, parity conditions, and international capital flows

What This Reading Covers

LM08 is the **calculation-heavy** backbone of the Economics topic. You will quote currency pairs, compute **cross rates** and bid-ask cross rates from two bilateral quotes, calculate **percentage appreciation/depreciation**, derive **forward rates** from points or CIP, and link spot rates, forwards, interest rates, inflation, and expected future spot rates through the **international parity conditions**. You will also see how the **FX carry trade** exploits (and risks) these relationships, and how capital flows and monetary policy drive exchange rates through the **Mundell-Fleming** and **portfolio-balance** frameworks.

Learning Outcome Statements

- LOS a** Calculate and interpret the percentage change in a currency relative to another currency
- LOS b** Calculate the result of a triangular arbitrage trade and a currency cross-rate
- LOS c** Explain spot and forward rates and calculate the forward premium/discount for a given currency
- LOS d** Calculate the mark-to-market value of a forward contract
- LOS e** Explain international parity conditions (covered interest rate parity, uncovered interest rate parity, forward rate parity, purchasing power parity, the international Fisher effect)
- LOS f** Describe relations among the international parity conditions
- LOS g** Evaluate the use of the current spot rate, the forward rate, purchasing power parity, and uncovered interest parity to forecast future spot exchange rates
- LOS h** Explain approaches to assessing the long-run fair value of an exchange rate
- LOS i** Describe the carry trade and its relation to uncovered interest rate parity and calculate the profit on a carry trade
- LOS j** Explain how flows in the balance of payment accounts affect currency exchange rates
- LOS k** Explain the Mundell-Fleming model, the monetary approach, and the portfolio-balance approach to exchange rate determination

Exam Tip: LM08 is the most formula-dense reading in Economics. Memorize: (1) cross-rate chaining ($A/B \times B/C = A/C$), (2) CIP formula $F = S \times (1 + i_P)/(1 + i_B)$, (3) the rule "higher-yield currency trades at a forward discount," and

(4) that percentage appreciation of X and percentage depreciation of Y are *not* equal in magnitude.

FOUNDATION · QUOTE CONVENTIONS

Reading an FX Quote

An exchange rate is the **price of one currency in terms of another**. On the CFA exam, quotes appear as **Price/Base** — read the "/" as "per." Example: **1.416 USD/EUR** means **each euro costs 1.416 U.S. dollars**.

Price vs. Base Currency

COMPONENT	MEANING	IN 1.416 USD/EUR
Price currency	The numerator — how much you pay	USD (1.416 dollars)
Base currency	The denominator — what you are buying (1 unit)	EUR (per 1 euro)
Direct quote	Domestic per foreign (from the price-currency country's perspective)	Direct for a U.S. investor
Indirect quote	Foreign per domestic (from the base-currency country's perspective)	Indirect for a eurozone investor

Inverting a Quote

To flip which currency is price vs. base, take the reciprocal. **1.416 USD/EUR** → $1/1.416 = 0.7062$ **EUR/USD**.

Spot vs. Forward

SPOT RATE

Exchange rate for **immediate delivery** (typically settle in T+2 for most pairs). Determined in the spot FX market.

FORWARD RATE

Exchange rate for a currency transaction to be done in the **future** (30, 60, 90, 180 days, 1 year). Locked in today, settled later.

Nominal vs. Real Exchange Rate

The **nominal** exchange rate is the stated market rate at a point in time. The **real** exchange rate adjusts for price-level differences between the two countries, capturing *relative purchasing power*:

$$Real(P/B) = Nominal(P/B) \times [CPI_{base}/CPI_{price}]$$

Interpretation: A decline in the real rate means that the purchasing power of the price currency — in terms of base-country goods — has increased.

Exam Tip: Always identify price vs. base **before** doing any calculation. An appreciation of the *base* means the quote rises; a depreciation of the *base* means the quote falls.

LOS A · PERCENTAGE CHANGE IN A CURRENCY

Percentage Appreciation and Depreciation

Rule: When the P/B quote *rises*, the **base currency appreciates** and the price currency depreciates. When the quote *falls*, the base currency depreciates and the price currency appreciates.

Base-Currency Percentage Change

The Asymmetry Problem

The percentage **appreciation** of currency X is *not* equal in magnitude to the percentage **depreciation** of currency Y. To find Y's depreciation, you must first invert the quote (make Y the base) and then compute the percentage change.

EXAMPLE 1 Asymmetry — USD/EUR 1.44 → 1.42

GIVEN

USD/EUR moves from **1.44** (initial) to **1.42** (later).

? FIND

(a) Percentage depreciation of the EUR (the base). (b) Percentage appreciation of the USD (the price currency).

→ SOLUTION

1

EUR is the base — apply the base-currency formula directly.

$$\% \Delta \text{ EUR} = 1.42 / 1.44 - 1 = -0.01389 = -1.39\%$$

The USD price of a euro went down → EUR depreciated 1.39%.

2

For the USD move, make USD the base — invert both quotes.

$$\text{Old EUR/USD} = 1 / 1.44 = 0.6944$$

$$\text{New EUR/USD} = 1 / 1.42 = 0.7042$$

- 3 Apply the base-currency formula to the inverted quotes.

$$\% \Delta \text{ USD} = 0.7042 / 0.6944 - 1 = +0.01408 = +1.41\%$$

- 4 Compare magnitudes.

$$| -1.39\% | \neq | +1.41\% |$$

✓ ANSWER

EUR depreciation -1.39%

USD appreciation +1.41%

◆ TAKEAWAY

The asymmetry arises because the two percentages use different denominators. On a CFA question that asks for "appreciation of the price currency" you **must** invert before computing — you cannot simply negate the base-currency percentage.

Exam Tip: Default formula: $\% \Delta \text{ base} = (\text{new} / \text{old}) - 1$. If the question asks about the *price* currency, invert first, then apply the same formula to the new (inverted) base.

LOS B · CURRENCY CROSS RATES

Calculating Cross Rates from Two Bilateral Quotes

A **cross rate** is the exchange rate between two currencies derived from their rates against a **common third currency** (usually the USD). Cross rates must satisfy **triangular no-arbitrage** — any mismatch creates a riskless profit opportunity.

The Chaining Rule

Line up the two bilateral quotes so that the common currency appears **once in a numerator** and **once in a denominator**, then multiply. The common currency cancels, leaving the desired cross.

$$(A/B) \times (B/C) = A/C$$

If a quote is oriented the wrong way, take its reciprocal first.

EXAMPLE 2 CAD/EUR from CAD/USD & USD/EUR

GIVEN

CAD/USD = 1.3980 · USD/EUR = 1.0950

? FIND

CAD/EUR cross rate.

→ SOLUTION

1

Align so USD cancels.

$$(CAD/USD) \times (USD/EUR) = CAD/EUR$$

2

Multiply.

$$1.3980 \times 1.0950 = 1.5308$$

✓ ANSWER

$$\text{CAD/EUR} = 1.5308$$

EXAMPLE 3 JPY/CAD — When One Quote Needs Inverting

☰ GIVEN

$$\text{CAD/USD} = 1.3980 \cdot \text{JPY/USD} = 121.10$$

? FIND

JPY/CAD cross rate.

→ SOLUTION

1

We need CAD in the denominator to end at JPY/CAD. Invert CAD/USD to get USD/CAD.

$$\text{USD/CAD} = 1 / 1.3980 = 0.7153$$

2

Chain through USD.

$$(\text{JPY/USD}) \times (\text{USD/CAD}) = \text{JPY/CAD}$$

3

Multiply.

$$121.10 \times 0.7153 = 86.62$$

✓ ANSWER

$$\text{JPY/CAD} = 86.62$$

◆ TAKEAWAY

Whenever the common currency appears in the "wrong place," invert one quote so it cancels. The final cross rate has the target pair P/B in place.

Four Cross Rates from One Three-Currency Setup (IFT practice)

Given: $\text{USD/EUR} = 1.3690$ · $\text{CHF/USD} = 0.9164$ · $\text{USD/GBP} = 1.5160$

CROSS	CHAIN	RESULT
CHF/EUR	$(\text{CHF/USD}) \times (\text{USD/EUR}) = 0.9164 \times 1.3690$	1.2546
GBP/EUR	$(1 / \text{USD/GBP}) \times (\text{USD/EUR}) = (1/1.5160) \times 1.3690$	0.9030
CHF/GBP	$(\text{CHF/USD}) \times (\text{USD/GBP}) = 0.9164 \times 1.5160$	1.3893

Exam Tip: If the chain doesn't cancel cleanly, invert one of the quotes. Any cross rate in the market that deviates from the computed no-arbitrage cross creates a triangular arbitrage opportunity.

LOS B · BID-ASK CROSS RATES

Bid-Ask Cross Rate Rules

Real quotes come with a **bid** (dealer buys the base) and an **ask/offer** (dealer sells the base). Crossing two bid-ask quotes is *not* a simple midpoint average — the rule depends on which side of the trade you, the client, are on.

The Core Principle

When constructing a cross rate A/C from two bilateral quotes A/B and B/C:

- 1 **To buy the base (C) / sell A:** use the *ask* at each leg that the dealer quotes. You want the **highest** A/C cross.
- 2 **To sell the base (C) / buy A:** use the *bid* at each leg. You want the **lowest** A/C cross.
- 3 When a quote must be inverted, **swap bid and ask:** bid of X/Y = 1/(ask of Y/X); ask of X/Y = 1/(bid of Y/X).

EXAMPLE 4 Cross-Rate Bid & Ask — Buying the Base

GIVEN

USD/EUR bid-ask = **1.3689 / 1.3691** · CHF/USD bid-ask = **0.9163 / 0.9165**

A client wants to **buy CHF and sell EUR** — i.e., trade at the cross rate CHF/EUR.

? FIND

The dealer's **ask** on the CHF/EUR cross (the rate at which the client will pay to buy CHF using EUR).

→ SOLUTION

- 1 Chain: $(\text{CHF/USD}) \times (\text{USD/EUR}) = \text{CHF/EUR}$.
- 2 To *buy CHF* (the first base in the chain), use the **ask on CHF/USD**. To *sell EUR* (pay the dealer at his offer), use the **ask on USD/EUR**.

$$\text{Ask}_{\text{CHF/EUR}} = \text{Ask}_{\text{CHF/USD}} \times \text{Ask}_{\text{USD/EUR}} = 0.9165 \times 1.3691$$

3

Multiply.

$$0.9165 \times 1.3691 = 1.2548$$

✓ ANSWER

Ask CHF/EUR = 1.2548

EXAMPLE 5

Cross-Rate Bid & Ask — Selling the Base

GIVEN

Same market: USD/EUR bid-ask = **1.3689 / 1.3691** · CHF/USD bid-ask = **0.9163 / 0.9165**

A client wants to **sell CHF and buy EUR** (the opposite trade).

? FIND

The dealer's **bid** on the CHF/EUR cross.

→ SOLUTION

1

Selling the base in the A/C cross → use the **bid** side at each leg.

$$\text{Bid}_{\text{CHF/EUR}} = \text{Bid}_{\text{CHF/USD}} \times \text{Bid}_{\text{USD/EUR}}$$

2

Plug in.

$$0.9163 \times 1.3689 = 1.2542$$

3

Combine with Example 4 for the full cross-rate quote.

$$\text{CHF/EUR bid-ask} = 1.2542 / 1.2548$$

✓ ANSWER

Bid (client sells base CHF → EUR)

1.2542

Ask (client buys base CHF ← EUR)

1.2548

◆ TAKEAWAY

Both sides of the cross are wider than the underlying spreads because the dealer charges at each leg. This is why cross-rate spreads are typically larger than single-pair spreads.

Quick Bid-Ask Memory Aid

"Same side both legs." If you are buying the base of the final cross, use ask×ask. If you are selling the base, use bid×bid. If a leg needs to be inverted, remember **1/ask becomes the new bid** (and 1/bid becomes the new ask).

QUIZ 1 · CHECKPOINT

Test Your Understanding: Quotes, % Changes & Cross Rates

Q1. The USD/EUR quote moves from 1.2500 to 1.2625. Which of the following is *most accurate*?

Q2. Given $\text{CAD/USD} = 1.3980$ and $\text{USD/EUR} = 1.0950$, the no-arbitrage CAD/EUR cross rate is *closest to*:

Q3. Given $\text{JPY/USD} = 121.10$ and $\text{CAD/USD} = 1.3980$, the JPY/CAD cross rate is *closest to*:

Q4. A client wants to buy CHF and sell EUR. CHF/USD bid-ask is 0.9163/0.9165, and USD/EUR bid-ask is 1.3689/1.3691. The dealer's ask on CHF/EUR is *closest to*:

LOS C · FORWARD QUOTATIONS

Forward Rates: Points & Percentages

Forward FX rates are typically quoted as **spot ± points (pips)**. Positive points ⇒ **forward premium** on the base currency. Negative points ⇒ **forward discount** on the base.

Converting Points to a Decimal

A point is **one unit in the last displayed digit** of the spot quote. For a 4-decimal quote (e.g. USD/EUR 1.4158), 1 point = 0.0001. Divide the point count by 10,000 to get the decimal adjustment.

$$Forwardrate = Spotrate \pm (points/10,000)$$

Example — Source Quote: USD/EUR 1.2875 with Pip Schedule

TENOR	POINTS	FORWARD RATE (USD/EUR)
Spot	—	1.2875
1 week	−0.3	1.28747
1 month	−1.1	1.28739
3 months	−5.5	1.28695
6 months	−13.3	1.28617
12 months	−26.5	1.28485

All negative points — the EUR is at a forward discount at every tenor. Points are also called *swap points*.

EXAMPLE 6 90-Day Forward from Points, 1-Year Forward from %

☰ GIVEN

USD/EUR spot = 1.4345 · 90-day forward quoted at **-50 points** · 1-year forward quoted at **-0.78%**

? FIND

The 90-day and 1-year all-in forward rates.

→ SOLUTION

1

Convert 90-day points to decimal.

$$-50 / 10,000 = -0.0050$$

2

Add to spot.

$$F(90) = 1.4345 + (-0.0050) = 1.4295 \text{ USD/EUR}$$

3

Apply the percentage quote.

$$F(1y) = \text{Spot} \times (1 + \% \text{ quote}) = 1.4345 \times (1 - 0.0078)$$

4

Evaluate.

$$= 1.4345 \times 0.9922 = 1.4233 \text{ USD/EUR}$$

✓ ANSWER

90-day forward

1.4295

1-year forward

1.4233

◆ TAKEAWAY

Both quotes are negative → EUR (the base) is at a forward discount at both horizons. This implies USD interest rates are *lower* than EUR interest rates (via CIP — see next step).

Forward Premium / Discount (Base Currency)

$$\text{Forward premium or discount} = (F - S)/S = F/S - 1$$

If $F > S \rightarrow$ base currency at a **forward premium**. If $F < S \rightarrow$ base at a **forward discount**. The magnitude of the premium/discount scales with the **interest-rate differential** and the **time to maturity**.

LOS C, E · COVERED INTEREST RATE PARITY

Covered Interest Rate Parity (CIP)

CIP is a no-arbitrage identity: any investor who borrows in one currency, converts at spot, invests at the foreign rate, and locks in the currency of conversion with a forward must earn exactly the domestic rate — otherwise riskless profit exists.

The No-Arbitrage Derivation (from source)

TWO EQUIVALENT WAYS TO END UP WITH USD AT TIME T_1

Path A: Invest Domestic
Invest 1 unit (d) for t at rate i_d
End value = $(1 + i_d)$

Path B: Convert & Invest Foreign
Convert 1(d) → $1/S(f/d)$ units of (f)
Invest at i_f , lock at $F(f/d)$
End value = $S(f/d) \times (1 + i_f) / F(f/d)$ in (d)

No-Arbitrage $\Rightarrow$ Path A = Path B
 $(1 + i_d) = S(f/d)(1 + i_f) / F(f/d)$

$F(f/d) = S(f/d) \times (1 + i_f) / (1 + i_d)$

CIP Formula (Exam Form)

$$F_{P/B} = S_{P/B} \times (1 + i_P) / (1 + i_B)$$

If annualized rates are given, de-annualize using $n/360$ (or the relevant day-count):

$$F_{P/B} = S_{P/B} \times [1 + i_P \times (n/360)] / [1 + i_B \times (n/360)]$$

EXAMPLE 7 CIP — 1-Year Forward Rate

GIVEN

$S_{f/d} = 1.6535$ · $i_d = 3.5\%$ · $i_f = 5.0\%$ · Horizon = 1 year

? FIND

The no-arbitrage forward rate $F_{f/d}$.

→ SOLUTION

1 Apply CIP with price = f, base = d.

$$F_{f/d} = S_{f/d} \times (1 + i_f) / (1 + i_d)$$

2 Plug in.

$$= 1.6535 \times (1.05) / (1.035)$$

3 Evaluate.

$$= 1.6535 \times 1.01449 = 1.6775$$

4 Compute forward points.

$$(1.6775 - 1.6535) \times 10,000 = +240 \text{ pips} = +2.4\phi$$

✓ ANSWER

$$F_{f/d} = 1.6775 (+240 \text{ pips})$$

◆ TAKEAWAY

Since $i_f (5\%) > i_d (3.5\%)$, the foreign (f) currency is higher-yielding and therefore trades at a **forward discount**. The forward rate $F_{f/d}$ rises from 1.6535 to 1.6775 — more units of f per d at the forward $\Rightarrow$ f has *depreciated* going forward.

The Golden Rule of Forward Premiums/Discounts

Higher-yielding currency always trades at a forward DISCOUNT; lower-yielding currency trades at a forward PREMIUM. Intuition: if this were not true, you could borrow the low-yield currency, invest in the high-yield currency, lock the FX with a forward, and earn a riskless spread.

EXAMPLE 8 Triangular / Covered Arbitrage When F Is Mispriced

GIVEN

Same setup as Example 7. The market quotes the 1-year $F_{f/d} = 1.6900$ (instead of the no-arb 1.6775).

? FIND

The arbitrage trade.

→ SOLUTION

- 1 Rule: **Buy low, sell high.** The market offers more f per d at t_1 than fair $\rightarrow$ (f) is "too cheap" at the forward. Alternatively, (d) is expensive in the forward.
- 2 Trade: **Borrow 1.6535 units of f**, convert to **1 unit of d** at spot, **invest d at i_d** , and **sell $F_{f/d}$ at 1.69** (lock to convert d $\rightarrow$ f at maturity).
- 3 Compare effective cost.

$$(1 + i_d) = 1.035 > 1.6535 \times 1.05 / 1.69 = 1.0273$$
- 4 Riskless profit: $1.035 - 1.0273 \approx 0.77\%$ per unit of d.

◆ TAKEAWAY

Whenever the market forward deviates from the CIP formula, borrow the currency where you can earn more via the synthetic path and invest in the currency where you pay less. Arbitrageurs collapse the mispricing back to parity.

LOS C, E · CIP WITH DAY-COUNT

CIP Practice — Day-Count Adjustments

EXAMPLE 9 1-Year Forward AUD/HKD via CIP

GIVEN

$S_{\text{AUD/HKD}} = 0.1526$ · 1-year HKD risk-free rate = 7% · 1-year AUD risk-free rate = 3%

? FIND

The 1-year no-arbitrage forward rate and the implied forward discount/premium on HKD.

→ SOLUTION

1 Apply CIP. Here AUD is price, HKD is base.

$$F_{\text{AUD/HKD}} = S_{\text{AUD/HKD}} \times (1 + i_{\text{AUD}}) / (1 + i_{\text{HKD}})$$

2 Plug in.

$$= 0.1526 \times (1.03) / (1.07) = 0.1526 \times 0.96262$$

3 Evaluate.

$$= 0.1469$$

4 Discount on HKD.

$$(0.1469 / 0.1526) - 1 = -3.74\%$$

5 Compare to interest-rate differential.

$$i_{\text{AUD}} - i_{\text{HKD}} = 3\% - 7\% = -4\%$$

✓ ANSWER

F(1y) AUD/HKD

0.1469

HKD forward discount

-3.74%

◆ TAKEAWAY

HKD (the higher-yielding currency, 7%) trades at a **forward discount**. The discount of -3.74% closely tracks the interest-rate differential of -4.00% — the two are equal when compounded exactly.

EXAMPLE 10

180-Day GBP/EUR Forward with n/360 De-Annualization

☰ GIVEN

$S_{\text{GBP/EUR}} = 0.8128$ · 180-day EUR LIBOR = **1.280%** (annualized) · 180-day GBP LIBOR = **1.175%** (annualized)

? FIND

The 180-day forward rate GBP/EUR and the forward points.

→ SOLUTION

1

With EUR as base, apply CIP with n/360 scaling.

$$F = S \times [1 + i_{\text{GBP}} \times (180/360)] / [1 + i_{\text{EUR}} \times (180/360)]$$

2

Numerator.

$$1 + 0.01175 \times 0.5 = 1.005875$$

3

Denominator.

$$1 + 0.01280 \times 0.5 = 1.006400$$

4

Ratio $\times$ spot.

$$F = 0.8128 \times (1.005875 / 1.006400) = 0.8128 \times 0.999478 = 0.8124$$

5

Forward points (4-decimal quote).

$$(0.8124 - 0.8128) \times 10,000 = -4 \text{ points}$$

✓ ANSWER

F(180d) GBP/EUR

0.8124

Forward points

-4

◆ TAKEAWAY

Because the EUR rate (1.280%) is higher than the GBP rate (1.175%), the EUR (base) must trade at a **forward discount** — so $F < S$. Percentage change in spot is approximately proportional to the **interest-rate differential** ($i_P - i_B$).

Exam Tip: On the exam, always check the tenor: if rates are "annualized" and tenor is 90/180-day, de-annualize as $i \times n/360$ before plugging into CIP. A common trap is to plug the annualized rates raw into a sub-year formula.

LOS E, F · INTERNATIONAL PARITY CONDITIONS

Uncovered IRP, PPP & the Fisher Effects

Uncovered Interest Rate Parity (UIP) replaces the locked-in forward rate with the *expected* future spot rate. It says investors should not, on average, earn excess returns from borrowing a low-yield currency and investing in a high-yield currency — because the high-yield currency is expected to depreciate by the interest-rate differential.

UIP — Formula

The expected percentage change in the P/B spot rate equals the interest-rate differential (price minus base). Equivalently:

$$E[S_{P/B}(t+1)] = S_{P/B}(t) \times (1 + i_P) / (1 + i_B).$$

EXAMPLE 11 UIP — Expected Future Spot Rate

GIVEN

$$S_{USD/EUR} = 1.2000 \cdot 1\text{-year } i_{USD} = 2\% \cdot 1\text{-year } i_{EUR} = 5\%$$

? FIND

The UIP-implied expected 1-year spot USD/EUR.

→ SOLUTION

1 Apply UIP.

$$E[S_{USD/EUR}] = 1.2000 \times (1.02) / (1.05)$$

2 Evaluate.

$$= 1.2000 \times 0.97143 = 1.1657$$

3 Check via approximation.

$$\% \Delta \approx i_{\text{USD}} - i_{\text{EUR}} = 2\% - 5\% = -3\% \rightarrow 1.2000 \times (1 - 0.03) = 1.1640 \text{ (close)}$$

✓ ANSWER

$$E[S_{\text{USD/EUR}}] \approx 1.1657$$

◆ TAKEAWAY

EUR is the higher-yielding currency (5%) and UIP predicts it will **depreciate** against USD by the interest-rate differential. This exactly offsets the higher EUR rate so no excess return is earned on average. **Empirically, UIP fails in the short run** — this is what makes the carry trade profitable on average.

Purchasing Power Parity (PPP)

Law of One Price	Identical goods should sell for the same price in different countries (when priced in the same currency).
Absolute PPP	The spot rate equals the ratio of the two countries' price levels: $S_{P/B} = \text{CPI}_P / \text{CPI}_B$. Rarely holds in practice — non-traded goods, trade barriers, tax differences.
Relative PPP	Percentage change in spot $\approx$ inflation differential: $\% \Delta S_{P/B} \approx \pi_P - \pi_B$. The country with higher inflation sees its currency depreciate.
Ex ante PPP	Expected percentage change in spot = expected inflation differential.

Fisher Effects

DOMESTIC FISHER EFFECT

Nominal interest rate $\approx$ real rate + expected inflation.

$$i \approx r + E(\pi)$$

INTERNATIONAL FISHER EFFECT

If real rates are equal across countries (Real Interest Rate Parity), the **nominal interest-rate differential equals the expected inflation differential**.

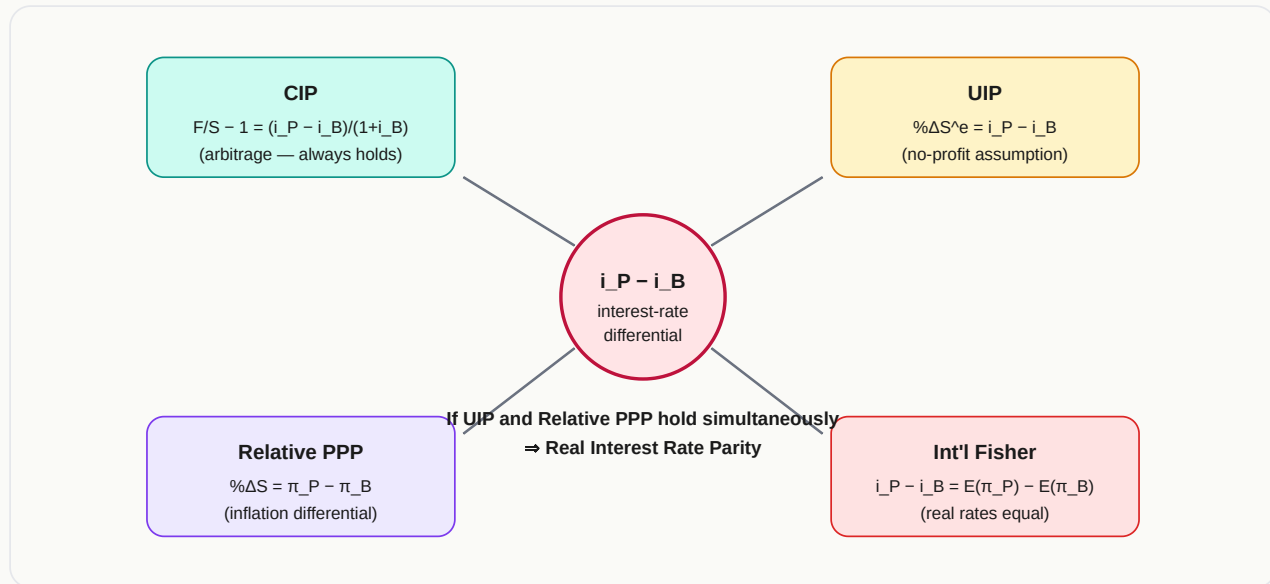
$$i_P - i_B \approx E(\pi_P) - E(\pi_B)$$

Exam Tip: The three differentials — **interest rate**, **expected inflation**, and **expected currency depreciation** — are all predicted to be equal under the full set of parity conditions. This is the key intuition behind LOS f.

LOS F · RELATIONS AMONG PARITY CONDITIONS

How the Parity Conditions Connect

THE PARITY "WHEEL" — FIVE IDENTITIES, ONE INTEREST-RATE DIFFERENTIAL



The Five Key Relationships

CONDITION	STATEMENT	ECONOMIC FORCE
CIP	$F_{P/B}/S_{P/B} - 1 \approx i_P - i_B$	Riskless arbitrage — holds essentially always in liquid currency markets.
UIP	$E(\% \Delta S_{P/B}) \approx i_P - i_B$	Assumption: investors are risk-neutral; no excess return is expected from cross-currency carry.
Forward-rate parity	$F_{P/B} = E(S_{P/B})$	Direct consequence of CIP + UIP holding together $\rightarrow$ forward is an unbiased predictor of future spot.
Relative PPP	$\% \Delta S_{P/B} \approx \pi_P - \pi_B$	Goods-market arbitrage; typically valid over long horizons only.
International Fisher	$i_P - i_B \approx E(\pi_P) - E(\pi_B)$	Real interest rates equal across countries (Real Interest Rate Parity).

Putting It Together

If **Relative PPP** + **International Fisher** + **UIP** all hold, then:

$$\% \Delta S^e = i_P - i_B = E(\pi_P) - E(\pi_B)$$

and the **forward rate is an unbiased predictor** of the future spot rate (forward rate parity).

Empirical Reality

CIP	Virtually always holds — this is an arbitrage relationship.
Relative PPP	Does not hold over short/medium horizons; approximately holds over very long horizons.
UIP	Fails over short horizons; the high-yield currency tends to <i>appreciate</i> (not depreciate), enabling profitable carry trades.
Forward rate parity	Interest rates are poor predictors of future spot rates — empirically, forward rates are biased predictors.
Real rate parity	Real rates differ across countries, especially over short/medium horizons.

QUIZ 2 · CHECKPOINT**Test Your Understanding: Forwards & Parity**

Q5. $S_{f/d} = 1.6535$, $i_d = 3.5\%$, $i_f = 5.0\%$. The 1-year no-arb forward $F_{f/d}$ is *closest to*:

Q6. If the 1-year AUD rate is 3% and 1-year HKD rate is 7%, the HKD (base) should trade at a 1-year:

Q7. USD/EUR spot is 1.4158. The 6-month forward quote is +25.3 points. The 6-month forward rate is:

Q8. If Relative PPP, UIP, and International Fisher all hold simultaneously, which is *most accurate*?

LOS I · FX CARRY TRADE

The FX Carry Trade

The **carry trade** is a speculative strategy that exploits the empirical *failure* of UIP: **borrow in a low-yielding currency, convert at spot, and invest in a high-yielding currency**. If UIP held perfectly, the expected return would be zero. In reality, high-yield currencies often fail to depreciate as much as UIP predicts — generating positive expected carry returns.

Mechanics — Three Steps

- 1 **Borrow** in the funding (low-yield) currency at i_F .
- 2 **Convert** to the investment (high-yield) currency at the current spot rate.
- 3 **Invest** at the higher rate i_H ; at horizon, convert back to funding currency at the future spot rate (*not* locked).

Profit Formula

$$\text{Carry} - \text{trade profit} \approx (i_H - i_F) -$$

If the high-yield currency depreciates exactly by the interest differential (UIP), profit = 0. If it depreciates less (or appreciates), profit is positive.

EXAMPLE 12

Carry Trade — JPY-Funded NZD Investment

GIVEN

Borrow JPY at $i_{JPY} = 0.5\%$ · Invest in NZD at $i_{NZD} = 5.0\%$ · Spot $S_{JPY/NZD} = 70.00$ · End-of-year spot $S_{JPY/NZD} = 68.50$ (NZD depreciates 2.14% against JPY)

? FIND

The 1-year return on a carry trade funded by borrowing 10 million JPY.

→ SOLUTION

- 1 Convert JPY to NZD at spot.

$$10,000,000 / 70 = 142,857.14 \text{ NZD}$$

- 2 Invest for 1 year at 5%.

$$142,857.14 \times 1.05 = 150,000 \text{ NZD}$$

- 3 Convert back at end-of-year spot 68.50.

$$150,000 \times 68.50 = 10,275,000 \text{ JPY}$$

- 4 Repay JPY loan with interest.

$$10,000,000 \times 1.005 = 10,050,000 \text{ JPY}$$

- 5 Profit.

$$10,275,000 - 10,050,000 = 225,000 \text{ JPY}$$

- 6 Return on borrowed principal.

$$225,000 / 10,000,000 = 2.25\%$$

- 7 Quick check via approximation.

$$(i_{\text{NZD}} - i_{\text{JPY}}) - \text{NZD depreciation} \approx (5\% - 0.5\%) - 2.14\% = 2.36\% \checkmark \text{ close}$$

✓ ANSWER

Carry return $\approx +2.25\%$

◆ TAKEAWAY

Under strict UIP, the NZD should have depreciated by ~4.5% (the interest differential) and the profit should have been zero. In this example NZD depreciated only 2.14%, leaving positive carry profit.

Risks — Why Carry Trades Blow Up

The return distribution of carry trades is **negatively skewed with high kurtosis** — long stretches of small positive returns punctuated by sharp losses. **"Picking up nickels in front of a steamroller."**

Funding-currency appreciation

In risk-off regimes, funding currencies (JPY, CHF) rally as traders unwind positions, crushing carry returns.

High-yield-currency crash

EM currencies with high nominal rates often crash during global risk aversion or domestic shocks.

Leverage amplification

Carry trades are typically heavily leveraged; small adverse moves in the spot wipe out equity quickly.

Liquidity risk

When everyone unwinds simultaneously, bid-ask spreads blow out and exit costs spike.

Exam Tip: The carry trade is **profitable only because UIP fails empirically**. The exam favorite is to ask which parity condition carry-trade profitability implies does not hold → answer: **uncovered interest rate parity**.

LOS J · BALANCE-OF-PAYMENTS FLOWS & CURRENCY VALUES

Capital Flows & Exchange Rates

A country's **balance of payments** must balance: an imbalance in goods/services trade (current account) is mirrored by an opposite imbalance in investment flows (capital account).

The National-Income Identity

$$X - M = (S - I) + (T - G)$$

$X - M$ Trade balance (current account)

$S - I$ Private savings minus investment

$T - G$ Government surplus (taxation – spending)

A **current account deficit** ($X - M < 0$) must be matched by a **capital account surplus** — the country imports capital to fund the excess consumption/investment.

Why Capital Flows Dominate Short-Term FX

Capital flows adjust **more rapidly** than savings and the price of goods. Hence, capital flows are the **primary determinant of short- and medium-term exchange rates**, while trade flows and PPP dominate only in the very long run.

Key Drivers of Capital Flows

DRIVER	EFFECT ON DOMESTIC CURRENCY
Higher domestic real interest rates	Attract foreign capital → currency <i>appreciates</i>
Expansionary monetary policy	Lower real rates → capital outflows → currency <i>depreciates</i>
Rising productivity / growth expectations	Attract equity inflows → currency appreciates
Risk-on global sentiment	EM / high-yield currencies appreciate; funding currencies (JPY, CHF) depreciate

Sustained current account deficit

Long-run depreciation if investor appetite for the currency wanes

Capital Flow Restrictions

Governments may impose capital flow restrictions to:

- 1 Reduce volatility of domestic asset prices caused by hot-money inflows/outflows
- 2 Maintain a fixed exchange rate peg
- 3 Keep domestic interest rates low
- 4 Protect strategic industries from foreign ownership

Exam Tip: On the exam, remember the dominance rule: **short-run FX = capital flows; long-run FX = trade flows and PPP**. This reverses the textbook intuition many students start with.

LOS K · EXCHANGE RATE DETERMINATION MODELS

Mundell-Fleming, Monetary & Portfolio-Balance Approaches

Three complementary frameworks explain how monetary and fiscal policy combine with exchange-rate regimes to move currencies.

1. Mundell-Fleming Model — Floating vs. Fixed

MUNDELL-FLEMING 4-QUADRANT ANALYSIS (SHORT-RUN, CAPITAL MOBILE)

FLOATING EXCHANGE RATE

Expansionary Monetary

$i \downarrow \rightarrow$ capital outflows
Currency DEPRECIATES
Expansionary fiscal: $i \uparrow$
Inflows $\rightarrow$ APPRECIATES

Policies push currency opposite directions

Opposite Combos

Tight M + Loose F: $\uparrow \uparrow$
 $\rightarrow$ strong appreciation
Loose M + Tight F: $\downarrow \downarrow$
 $\rightarrow$ strong depreciation

FIXED EXCHANGE RATE (peg)

Mundell Trilemma

Can have only 2 of 3:
• Fixed exchange rate
• Free capital flows
• Independent monetary policy

Under a peg, monetary policy loses independence

Fiscal Policy Under Peg

Capital mobility + peg $\Rightarrow$
fiscal policy VERY effective
(central bank must sterilize
to defend the peg)

Opposite conclusion vs. floating

Mundell-Fleming Summary Table

	EXPANSIONARY MONETARY	EXPANSIONARY FISCAL
Floating + high capital mobility	Domestic currency DEPRECIATES (lower $i \rightarrow$ outflows)	Domestic currency APPRECIATES (higher $i \rightarrow$ inflows)
Floating + low capital mobility	DEPRECIATES (trade channel dominates: lower $i \rightarrow$ higher growth $\rightarrow$ higher imports)	DEPRECIATES (expanding demand raises imports more than capital inflows)
Fixed peg + high capital mobility	Monetary policy loses independence — defends peg	Fiscal policy is very effective; central bank must sterilize

Mundell Trilemma

A country can choose **only two** of: (1) fixed exchange rate, (2) free capital mobility, (3) independent monetary policy. Hong Kong's currency board chose (1) and (2) — giving up monetary independence. A floating rate (like the USD) gives up (1) to preserve (2) and (3).

2. Monetary Approach

A pure **monetarist** model ignores output effects and says the exchange rate is determined entirely by relative money supplies and relative price levels (PPP). An **increase in the domestic money supply** relative to foreign raises domestic prices → domestic currency **depreciates** by the inflation differential.

Dornbusch's sticky-price variant adds short-run **overshooting**: because goods prices adjust slowly but asset prices adjust instantly, an expansionary monetary shock causes the currency to *overshoot* its long-run depreciation in the short run.

3. Portfolio-Balance Approach (Fiscal Policy Long Run)

The portfolio-balance model focuses on **sustained fiscal imbalances**. A persistent fiscal expansion increases the supply of government debt. Global investors will hold the extra debt only if they are compensated — either higher yields or currency depreciation.

- 1 **Short run** (Mundell-Fleming): higher rates attract capital → currency appreciates.
- 2 **Long run** (portfolio-balance): investors demand lower currency value to absorb the expanding debt stock → currency **depreciates**.

This explains why persistent fiscal deficits eventually weigh on a currency despite short-run appreciation.

Exam Tip: The Mundell-Fleming model gives the *short-run* answer; the portfolio-balance model gives the *long-run* answer. On exam questions about "sustained" or "persistent" fiscal policy, default to the portfolio-balance conclusion (depreciation).

QUIZ 3 · CHECKPOINT

Test Your Understanding: Carry Trade, Flows & Mundell-Fleming

Q9. The carry trade is *most likely* profitable because:

Q10. An investor borrows 10,000,000 JPY at 0.5%, converts at $\text{JPY/NZD} = 70$, invests in NZD at 5%, and converts back at $\text{JPY/NZD} = 68.50$ one year later. The carry profit is *closest to*:

Q11. Under the Mundell-Fleming model with floating exchange rates and high capital mobility, expansionary fiscal policy is *most likely* to cause the domestic currency to:

Q12. Under the Mundell Trilemma, a country with a fixed exchange rate and free capital mobility must give up:

Q13. The *short-run* primary determinant of exchange rates is *most likely*:

Q14. The *portfolio-balance* approach predicts that *sustained* expansionary fiscal policy will *most likely*:

MODULE COMPLETE

Your Scorecard

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LM08 Key Takeaways — Exam-Ready Cheat Sheet

Quoting & Cross Rates

- 1 **Quote convention:** P/B = units of P per 1 unit of B. Rise in quote $\Rightarrow$ base appreciates, price depreciates. **Direct** = domestic/foreign; **Indirect** = foreign/domestic.
- 2 **Percentage change:** $\% \Delta \text{ base} = \text{new/old} - 1$. Asymmetric — the opposite-base $\% \Delta$ differs slightly; always invert before computing the other currency's change.
- 3 **Cross rate:** chain so common currency cancels: $(A/B)(B/C) = A/C$. Invert any leg oriented the wrong way.
- 4 **Bid-ask cross:** buying the base of the cross $\Rightarrow \text{ask} \times \text{ask}$; selling $\Rightarrow \text{bid} \times \text{bid}$. On inversion, $1/\text{ask}$ becomes new bid.

Forwards & CIP

- 5 **Points:** 1 point = 10^{-n} where n = last displayed digit. $F = S + (\text{points}/10,000)$ for 4-decimal quotes; or $F = S(1 + \%)$.
- 6 **CIP:** $F_{P/B} = S_{P/B} \times (1 + i_P)/(1 + i_B)$. For sub-year tenors, de-annualize with $n/360$.
- 7 **Golden rule:** higher-yield currency trades at a forward *discount*; lower-yield at a *premium*.

Parity Conditions

- 8 **UIP:** $E(\% \Delta S_{P/B}) \approx i_P - i_B$. Fails empirically $\Rightarrow$ carry trade profitability.

- 9 Relative PPP:** $\% \Delta S_{P/B} \approx \pi_P - \pi_B$. **International Fisher:** $i_P - i_B = E(\pi_P) - E(\pi_B)$. **Forward rate parity:** $F = E(S)$, implying the forward is an unbiased predictor.
- 10 Carry trade:** borrow low-yield, invest high-yield. Return $\approx (i_H - i_F) - \text{high-yield depreciation}$.
Negative skew, high kurtosis.

Capital Flows & Models

- 11 Balance of payments:** $X - M = (S - I) + (T - G)$. Current account deficits are funded by capital account surpluses.
- 12 Short run:** capital flows dominate FX. **Long run:** trade flows and PPP dominate.
- 13 Mundell-Fleming (floating, high mobility):** expansionary monetary $\Rightarrow$ currency depreciates; expansionary fiscal $\Rightarrow$ currency appreciates.
- 14 Portfolio balance (long run):** sustained fiscal deficits $\Rightarrow$ currency depreciation. **Mundell Trilemma:** fixed FX + free capital $\Rightarrow$ no monetary independence.

Last-Minute Exam Checklist

Cross rate: $(A/B)(B/C) = A/C$ · **Invert** if orientation wrong

CIP: $F = S \times (1 + i_P)/(1 + i_B)$ · sub-year: multiply rates by $n/360$

High yield $\Rightarrow$ forward discount · Points/10,000 = decimal

$\% \Delta$ appreciation $\neq$ $\% \Delta$ depreciation (always invert first)

Carry: borrow low, invest high; profits $\Rightarrow$ UIP fails

Short-run FX: capital flows · **Long-run FX:** PPP

Floating + mobile capital: loose monetary = depreciate; loose fiscal = appreciate

Trilemma: pick only 2 of {fixed FX, free capital, monetary independence}