LM06 · FIXED INCOME · CFA LEVEL I

Fixed-Income Bond Valuation — Prices and Yields

The PV formula, flat vs. full price, the four price–yield relationships, and matrix pricing.

Learning Outcomes

LOS a Calculate a bond's price given a YTM on or between coupon dates

LOS b Identify the relationships among price, coupon, maturity, and YTM

LOS c Describe matrix pricing

LOS A · BOND PRICE AS PV OF CASH FLOWS

A Bond's Price Is the PV of All Future Cash Flows

$$\mathrm{PV} = \frac{\mathrm{PMT}}{(1+r)^1} + \frac{\mathrm{PMT}}{(1+r)^2} + \ldots + \frac{\mathrm{PMT}+\mathrm{FV}}{(1+r)^N}$$

PV	bond price (present value)	PMT	periodic coupon payment	FV	face / par value at maturity
r	market discount rate (YTM per period)	N	number of periods to maturity		

The market discount rate r is the rate of return required by investors given the risk of the bond. If spot rates differ by maturity, discount each cash flow by its own spot rate Z_t:

$$\mathrm{PV} = \frac{\mathrm{PMT}}{(1+Z_1)^1} + \frac{\mathrm{PMT}}{(1+Z_2)^2} + \ldots + \frac{\mathrm{PMT}+\mathrm{FV}}{(1+Z_N)^N}$$

WORKED EXAMPLE · PRICING A 3-YEAR BOND

Worked Example — Pricing a Bond with Spot Rates

EXAMPLE **3-year, 5% annual-pay, $1,000 par bond**

GIVEN

Spot rates: 1yr = 4%, 2yr = 4.25%, 3yr = 4.5%. Coupon = 5%, annual pay. Par = $1,000.

SOLUTION

$$\text{Bond value} = \frac{50}{1.04} + \frac{50}{1.0425^2} + \frac{1{,}050}{1.045^3} = \mathbf{\$1{,}014.19}$$

ANSWER

$1,014.19 — priced at a premium because the coupon (5%) exceeds the average spot rate.

LOS A · BETWEEN COUPON DATES

Flat Price vs. Full Price (Accrued Interest)

Between coupon dates, the **full price** of a bond includes the **flat price** plus **accrued interest**.

- **Flat (clean) prices** are quoted in the market, to avoid artificial price swings from coupon accruals.
- **Accrued interest** = (t / T) × PMT, where t is days since last coupon and T is the coupon period in days.

$$\text{Full (dirty) price} = \text{Flat price} + \text{Accrued interest}$$
$$\text{PV}^{\text{full}} = \text{PV} \times (1 + r)^{t/T}$$

t	days since last coupon	T	total days in coupon period
r	YTM per coupon period		

EXAMPLE **Flat vs. full price at settlement**

GIVEN

$1,000 face value, semi - annual coupon bond, quoted price 103, accrued interest since last coupon =$25.

SOLUTION

$$\text{Flat price} = \$1{,}000 \times 103\% = \$1{,}030$$
$$\text{Full price} = \$1{,}030 + \$25 = \mathbf{\$1{,}055}$$

ANSWER

Seller receives **$1,055** at settlement.

LOS B · INVERSE, CONVEXITY, COUPON, MATURITY EFFECTS

The Four Relationships

EFFECT	STATEMENT
Inverse effect	Bond price is inversely related to the discount rate.
Convexity effect	For the same bond, the percentage price change is **greater** when the discount rate goes **down** than when it goes up (price–yield curve is convex).
Coupon effect	For the same maturity and same Δrate: prices of **low-coupon bonds change more** (greater %) than high-coupon bonds.
Maturity effect	For the same coupon and same Δrate: prices of **long-term bonds change more** than short-term bonds.

Summary: Low-coupon and **long-term** bonds are the most sensitive to discount-rate changes.

CHECKPOINT 1

Quiz 1: Pricing & Price-Yield

Answer all questions to test your understanding. Your score updates on the final step.

Q1. Using the spot rates 4% (1y), 4.25% (2y), 4.5% (3y), the price of a 3-year 5% annual-pay, $1,000 par bond is:

Q2. A bond's **flat (clean) price**:

Q3. The **convexity effect** states that for the same bond:

Q4. The **coupon effect** implies that, holding maturity constant, for a given yield change:

Q5. The **maturity effect** implies that, holding coupon constant, for a given yield change:

LOS C · MATRIX PRICING

Matrix Pricing — Valuing Illiquid Bonds

A method used to value **illiquid bonds** by using prices and yields on **comparable securities**. Comparables have the **same or similar credit risk, coupon rate, and maturity**.

Steps

1. Determine the YTM of comparable bonds.
2. Determine the average yields of different comparable bonds (e.g., by maturity bucket).
3. Calculate the market discount rate using **linear interpolation** across maturities.
4. Using the discount rate, compute the price of the target bond.

Exam Tip: Matrix pricing also provides a reasonable estimate of **required yield spread** for a new issue with no traded history.

VISUALIZATION · CONVEXITY

Visualizing the Price-Yield Relationship

The price-yield curve of an option-free bond is:

- **Downward sloping** — inverse relationship.
- **Convex** — curved toward the origin.

As a result of convexity, a duration-only estimate of price change **overstates** the price drop from a yield increase and **understates** the price rise from a yield drop. Adding the convexity adjustment improves the estimate (covered in LM12).

CHECKPOINT 2

Quiz 2: Full Price & Matrix Pricing

Answer all questions to test your understanding. Your score updates on the final step.

Q6. A $1,000face-valuebondwithaquotedpriceof102.5andaccruedinterestof$18 has a full price of:

Q7. Matrix pricing uses comparable-bond yields together with **linear interpolation** to *most directly* estimate:

Q8. Between coupon dates, accrued interest is *most accurately*:

Q9. The price-yield relationship for an option-free fixed-rate bond is:

Q10. Matrix pricing is *most commonly* applied to:

REVIEW · PRICING DECISION TREE

Bond Pricing Decision Tree

SITUATION	APPROACH
Single YTM and coupon dates	PV at single r using the standard PV formula
Spot rates known	PV of each cash flow at its own spot rate Z_t
Between coupon dates	Full price = PV × (1+r)^(t/T); subtract accrued to quote flat
Illiquid target bond	Matrix pricing: comparables → interpolated rate → PV

CHECKPOINT 3 · SYNTHESIS

Quiz 3: Synthesis

Answer all questions to test your understanding. Your score updates on the final step.

Q11. A 3-year, 6% annual-pay, $1,000 par bond priced at$1,028.39 implies a YTM of *approximately*:

Q12. Holding maturity constant, a zero-coupon bond is *most accurately*:

Q13. A bond trading at 97 is a **discount** bond. Its yield is:

Q14. Accrued interest is *most accurately* used to:

Q15. A flat (upward- or downward-moving) price-yield curve would *most likely* imply:

MODULE COMPLETE

Your Scorecard

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LM06 Key Takeaways — Exam-Ready Cheat Sheet

Core pricing

PV = sum of PMT/(1+r)^t + FV/(1+r)^N. With spot rates, discount each CF at its own Z_t. Full = Flat + Accrued (t/T × PMT).

Price-yield relationships

Inverse, **convex**; low-coupon & long-maturity = most sensitive (coupon + maturity effects).

Matrix pricing

Use comparables (similar credit, coupon, maturity) → average YTMs → linear interpolate → price illiquid target.

Exam memorized example: 3y 5% annual bond with spots 4/4.25/4.5 → $1,014.19.Quoted103+$25 accrued = \$1,055 full price.

Step 1 of 12