

LM09 · FIXED INCOME · CFA LEVEL I

The Term Structure of Interest Rates — Spot, Par, and Forward Curves

Three curves, one term structure: spot rates (yields on zeros), par rates (yields on at-par bonds), and forward rates (implied future short rates).

Learning Outcomes

LOS a Describe and compare spot, par, and forward curves; use each to value bonds

LOS b Define maturity structure and explain relationships among the three curves

LOS A · SPOT RATES & PRICING

Spot Rates

Spot rates are yields to maturity on **zero-coupon** bonds maturing at the date of each cash flow. No intermediate cash flows means the actual yield on the zero is used as the discount rate for that maturity.

$$PV = PMT/(1 + Z_1)^1 + PMT/(1 + Z_2)^2 + \dots + (PMT + FV)/(1 + Z_N)^n$$

Upward-sloping spot curves are typical under normal market conditions (longer-term zeros earn higher yields than shorter-term zeros). Unusual circumstances can produce flat or downward-sloping curves.

WORKED EXAMPLE · 3-YEAR BOND VIA SPOTS

Worked Example — Spot-Rate Pricing**EXAMPLE****3-year 5% annual-pay bond via spots 2% / 3% / 4%****SOLUTION**

$$PV = 5/1.02 + 5/1.03^2 + 105/1.04^3 = \mathbf{102.96}$$

Treat the bond as a portfolio of three zero-coupon bonds with 1, 2, and 3-year maturities at yields 2%, 3%, 4%.

Implied YTM: $102.96 = 5/(1+r) + 5/(1+r)^2 + 105/(1+r)^3 \Rightarrow r \approx \mathbf{3.93\%}$.

LOS A · PAR RATES

Par Rates

A **par rate** is the market discount rate for a specific maturity that results in a bond being priced at par. Par rates are derived from spot rates up to and including the maturity date.

$$100 = \text{PMT}/(1 + Z_1)^1 + \text{PMT}/(1 + Z_2)^2 + \dots + (\text{PMT} + 100)/(1 + Z_N)^N \text{ Solve for PMT} \rightarrow$$

Worked Example — Par Rates from Spots

EXAMPLE
Par rates from 4-year spot curve
GIVEN SPOTS (EAR)

1y 5.263%, 2y 5.616%, 3y 6.359%, 4y 7.008%.

SOLUTION

1y par rate: $100 = (\text{PMT} + 100)/1.05263 \rightarrow \text{PMT} = 5.263\%$.

2y par rate: $100 = \text{PMT}/1.05263 + (\text{PMT} + 100)/1.05616^2 \rightarrow \text{PMT} = 5.606\%$.

3y par rate: **6.306%**. 4y par rate: **6.899%**.

LOS A · FORWARD RATES

Forward Rates

A **forward rate** is a lending/borrowing rate for a **short-term loan to be made in the future**. Implied spot rates can be calculated as geometric averages of forward rates; the relationship is reversible.

$$(1 + Z_n)^n = (1 + Z_m)^m \times (1 + {}^m_2y(n - m))^{{}^m_2y(n - m)}$$

EXAMPLE**1-year forward rate two years from today****GIVEN**

3-year spot = 9.25%, 2-year spot = 9%.

SOLUTION

$$(1.0925)^3 = (1.09)^2 \times (1 + {}^2_1y) \Rightarrow {}^2_1y = (1.0925)^3 / (1.09)^2 - 1 = \mathbf{9.75\%}$$

Quick-approx: ${}^2_1y \approx 3(9.25\%) - 2(9\%) = 9.75\%$.

LOS A · PRICING VIA FORWARD RATES

Pricing with Forward Rates

Discount each cash flow by the **product of forward rates** through its period.

EXAMPLE 3-year 6% annual-pay \$1,000 par bond**GIVEN**

Current spot = 4%. 1y forward 1y from now = 5%. 1y forward 2y from now = 6%.

SOLUTION

$$V = 60/1.04 + 60/(1.04 \times 1.05) + 1,060/(1.04 \times 1.05 \times 1.06) = \mathbf{\$1,028.39}$$

CHECKPOINT 1

Quiz 1: Spot & Par

Answer all questions to test your understanding. Your score updates on the final step.

Q1. Given spots 2% (1y), 3% (2y), 4% (3y), a 3-year 5% annual-pay \$100 par bond is priced at:

Q2. The 2-year par rate derived from spots 5.263% (1y) and 5.616% (2y) equals:

Q3. Par rates are derived from:

Q4. When the spot curve is upward sloping, par rates are *typically*:

Q5. Implied spot rates are *best* computed from forwards via:

LOS B · RELATIONSHIPS AMONG CURVES

Relationships Among Spot, Par, and Forward Curves

Since par and forward rates can be derived from spot rates, the shape of the spot curve directly determines the shape of the other curves.

SPOT CURVE SHAPE	PAR RATES VS. SPOTS	FORWARD RATES VS. SPOTS
Upward sloping	Near, but below spots	Greater than spots
Flat	Equal to spots	Equal to spots
Downward sloping	Close to but above spots	Lower than spots

Quick mnemonic: forward rates are the marginal rates — they push or pull spots and pars away from parallel.

SUPPLEMENTAL · PAR / SWAP CURVE

Par Rates and Swap Curves

The **par curve** is commonly used as the reference for the **swap curve**. Market participants often quote swap rates (par-curve-equivalents) in major currencies; I-spread uses this curve.

Par rates also approximate the coupon an issuer would need to set on a new at-par issue at each maturity — a practical tool for primary pricing.

CHECKPOINT 2

Quiz 2: Forward Rates

Answer all questions to test your understanding. Your score updates on the final step.

Q6. If the 2-year spot rate is 9% and the 3-year spot rate is 9.25%, the 1-year forward rate two years out (2y1y) is *approximately*:

Q7. A 3-year 6% annual-pay \$1,000 par bond with spot = 4%, 1y forward 1y = 5%, 1y forward 2y = 6% is priced at:

Q8. An upward-sloping spot curve *most likely* implies a set of forward rates that are:

Q9. A flat spot curve implies that the par curve and forward curve are:

Q10. A forward rate is *best* described as:

REVIEW · SUMMARY MATRIX

Summary Matrix — Three Curves

CURVE	WHAT IT IS	USE IT TO PRICE
Spot	YTM on zeros	Any bond (PV by individual spots)
Par	Coupon rate that prices at par at each maturity	Reference for swap-curve-style pricing
Forward	Implied future short-term rates	Price via forward-rate product

CHECKPOINT 3 · SYNTHESIS

Quiz 3: Synthesis

Answer all questions to test your understanding. Your score updates on the final step.

Q11. For an upward-sloping spot curve, which ordering is correct?

Q12. Using spots 4% (1y), 4.25% (2y), 4.5% (3y), a 3y 5% annual-pay \$1,000 par bond prices at:

Q13. If two curves are identical (flat spot), the third (forward) is also flat and equal. This reflects:

Q14. The fastest way to price a 2y bond when the 1y spot and the 1y1y forward are known is:

Q15. The **swap curve** in a currency most closely relates to the:

MODULE COMPLETE

Your Scorecard

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LM09 Key Takeaways — Exam-Ready Cheat Sheet

Three curves

Spot = YTM on zeros; **par** = coupon that prices bond at par; **forward** = implied future short rate. All three are algebraically linked.

Arbitrage-free pricing

Any bond can be priced by spots directly, by discount-factor products of forwards, or (indirectly) by par rates & no-arbitrage logic.

Shape: Upward → forward > spot > par. Flat → all equal. Downward → par > spot > forward.

Curriculum numbers: 3y bond 5%, spots 4/4.25/4.5 → 1,014.19.3y61,028.39. 2y1y from 2y 9% & 3y 9.25% = 9.75%.