LM11 · FIXED INCOME · CFA LEVEL I

Yield-Based Bond Duration Measures and Properties

Macaulay, modified, money duration, and PVBP — how a bond's maturity, coupon, and yield level determine its interest-rate risk.

Learning Outcomes

LOS a Define, calculate, and interpret modified duration, money duration, and PVBP

LOS b Explain how a bond's maturity, coupon, and yield level affect its interest-rate risk

Exam Tip: Memorize: ModDur = MacDur / (1 + r); $\%\Delta PV_{full} \approx -\text{ModDur} \times \Delta y$; MoneyDur = AnnModDur × PV_{full}; PVBP = (PV– – PV+) / 2.

LOS A · YIELD DURATION AND CURVE DURATION

Two Categories of Duration

Yield duration

Bond's price sensitivity to a change in its **own YTM**. Assumes underlying cash flows are certain. Includes Macaulay, modified, money duration, and PVBP.

Curve duration

Bond's price sensitivity to a change in a **benchmark yield curve**. Accounts for the possibility of default or embedded options (covered in LM13).

Macaulay duration (MacDur)

The **weighted average of the time to receipt** of coupon interest and principal payments. Weights are the PV of each cash flow divided by PV_{full}. For a zero-coupon bond, MacDur = time to maturity. For a perpetuity, MacDur = (1 + r) / r.

LOS A · MODIFIED DURATION

Modified Duration

Modified duration is a **linear estimate of the %-price change** in a bond for a 100 bp change in YTM.

$$ModDur = MacDur/(1 + r)$$

EXAMPLE

ModDur from MacDur

GIVEN

2-year annual-pay $100 bond, MacDur = 1.87 years, YTM = 5%.

SOLUTION

$$ModDur = 1.87/1.05 = \mathbf{1.78}$$

For a 1% increase in YTM: %Δprice = –1.78 × 0.01 × 100 = **–1.78%**.

LOS A · APPROXMODDUR

Approximate Modified Duration

Estimates the slope of the tangent to the price-yield curve:

$$ApproxModDur = (PV_{-} - PV_{+})/(2\ \times\ \Delta yield\ \times\ PV_0)$$

EXAMPLE **12% 10-year par bond, 10 bp shock**

GIVEN

12% annual-pay bond, 10 years, trading at par. Shock ΔYTM = 10 bp = 0.001. PV_0 = 100.

SOLUTION

PV_: N=10, PMT=12, FV=100, I/Y=11.9, CPT PV = **100.57**.

PV_+: I/Y=12.1, CPT PV = **99.44**.

$$ApproxModDur = (100.57 - 99.44)/(2\ \times\ 100\ \times\ 0.001) = \mathbf{5.65}$$

CHECKPOINT 1

Quiz 1: MacDur & ModDur

Answer all questions to test your understanding. Your score updates on the final step.

Q1. A 5-year 3.2% semi-annual coupon bond at par has a MacDur of 9.3203 (semi-annual periods). At a periodic YTM of 1.6%, its annualized modified duration is *closest* to:

Q2. Given ApproxModDur = 5.65 and a YTM increase of 75 bp, the expected percentage price change is:

Q3. For a **zero-coupon bond**, Macaulay duration is *best* described as:

Q4. Yield duration assumes:

Q5. Macaulay duration of a perpetuity is:

LOS A · MONEY DURATION AND PVBP

Money Duration

Money duration (also called **dollar duration** in the US) is the money change in a bond's price for a given change in yield.

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MoneyDur = AnnModDur \; \times \; PV_{full} $\Delta PV_{full} = -
MoneyDur \; \times \; \Delta yield
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Price Value of a Basis Point (PVBP)

The **money change** in the full price of a bond when its YTM changes by **1 bp** (0.01%). Also called PV01 or DV01.

$$PVBP = (PV_{-} - PV_{+})/2BPV = ModDur \times PV_{full} \times 0.0001$$

EXAMPLE **PVBP on a 6-year 4% bond at YTM = 5%**

GIVEN

6-year, 4% annual coupon, YTM = 5%, price = 94.9243.

SOLUTION

PV_: N = 6, I/Y = 4.99, PMT = 4, FV = 100. CPT PV = **$94.9735**.

PV_{+}: N = 6, I/Y = 5.01, PMT = 4, FV = 100. CPT PV = **$94.8752**.

$$PVBP = (94.9735 - 94.8752)/2 = \mathbf{0.049}$$

LOS B · MATURITY, COUPON, YIELD EFFECTS

How Maturity, Coupon, and Yield Affect Duration

The interest-rate risk of a fixed-rate bond, measured by duration, moves in predictable directions. *All else equal:*

FACTOR	CHANGE	EFFECT ON DURATION
Maturity	Longer	**Increases** (usually)
Coupon rate	Higher	**Decreases**
Yield to maturity	Higher	**Decreases**

Higher coupon → more value delivered sooner → less sensitive to yield changes. Higher YTM → price-yield curve is flatter → less sensitive.

Special cases

- **Zero-coupon:** MacDur = time to maturity.
- **Perpetuity:** MacDur = (1 + r) / r (finite despite infinite cash flows).
- **FRN:** $\text{MacDur}_{\text{floater}} = (T - t) / T$, where T is the reset period and t is time since last reset. FRN duration is *small* and resets each period.
- **Discount coupon bond (very long maturity):** MacDur can actually *decrease* with longer maturity. That is why we qualify the maturity rule with “usually.”

KEY CONCEPT · DURATION GAP

Duration Gap and Investor Horizon

The **duration gap** is the bond's MacDur minus the investor's horizon.

$$Duration gap = MacDur - Investment horizon$$

Positive duration gap (MacDur > horizon)

Market price risk dominates. Investor is hurt by rising interest rates.

Negative duration gap (MacDur < horizon)

Coupon reinvestment risk dominates. Investor is hurt by falling interest rates.

EXAMPLE **Duration gap (MacDur = 7.8016, horizon = 8 years)**

SOLUTION

Duration gap = 7.8016 – 8 = **–0.1984**. Gap is **negative**.

A negative gap entails the risk of **lower** interest rates (reinvestment risk dominates). The loss from reinvesting coupons at a rate lower than 6% is larger than the gain from selling the bond at a price above the constant-yield price trajectory.

CHECKPOINT 2

Quiz 2: Money Duration & Properties

Answer all questions to test your understanding. Your score updates on the final step.

Q6. PVBP of a bond most accurately equals:

Q7. All else equal, modified duration is *smallest* for a bond with:

Q8. An investor has an 8-year horizon and buys a bond with MacDur = 7.8016. The duration gap of –0.1984 indicates:

Q9. A floating-rate note resets every 180 days. 56 days after the last reset, MacDur of the FRN is:

Q10. Money duration of a \$10 million par, AnnModDur = 6.2, PV_{full} = \$10.4 million bond is:

CONTEXT · SOURCES OF RETURN ON A BOND

Three Sources of Return

1. Coupon and principal payments.
2. Interest earned on **reinvested coupon payments**.
3. Capital gain or loss if sold before maturity.

If rates rise shortly after purchase: the seller loses from a lower sale price but gains on coupon reinvestment. If rates fall: the seller gains on the sale price but loses on reinvestment. These two risks offset exactly at the bond's **Macaulay duration** — that is the horizon at which a buy-and-hold investor realizes the YTM they locked in at purchase.

CHECKPOINT 3 · SYNTHESIS

Quiz 3: Synthesis

Answer all questions to test your understanding. Your score updates on the final step.

Q11. A bond's annualized modified duration is 4.587 and PV_{full} is $100. A 50 bp increase in YTM would change price by approximately:

Q12. At higher YTMs, ModDur is *smaller* because:

Q13. An *increase* in coupon rate *most likely*:

Q14. PVBP is used to:

Q15. A zero-coupon bond with a 10-year maturity at YTM = 5% has a modified duration of approximately:

MODULE COMPLETE

Your Scorecard

0 / 15

LM11 Key Takeaways — Exam-Ready Cheat Sheet

Core formulas

$$ModDur = MacDur/(1 + r)$$

Special cases

ZCB: MacDur = time to maturity. Perpetuity: MacDur = (1 + r)/r. FRN: $\text{MacDur}_{\text{floater}}$ = (T – t)/T.

Properties (all else equal)

Duration **increases** with longer maturity (usually), **decreases** with higher coupon, **decreases** with higher YTM.

Duration gap

Positive gap (MacDur > horizon) = market-price risk dominates (risk of rising rates). Negative gap = reinvestment risk dominates (risk of falling rates).

Step 1 of 12