LM12 · FIXED INCOME · CFA LEVEL I

Yield-Based Bond Convexity and Portfolio Properties

Convexity, the second-order correction to duration, plus how duration and convexity aggregate across a bond portfolio.

Learning Outcomes

LOS a Calculate and interpret convexity and the convexity adjustment

LOS b Calculate the %-price change of a bond given its duration and convexity

LOS c Calculate portfolio duration and convexity and explain limitations

Exam Tip: Always write the full formula: $\%\Delta PV_{full} = (-\text{AnnModDur} \times \Delta y) + (\tfrac{1}{2} \times \text{AnnConvexity} \times \Delta y^2)$. Convexity adjustment is **always positive** for an option-free bond.

LOS A · CONVEXITY

Convexity as the Second-Order Correction

Modified duration is a **linear** approximation of the price-yield relationship, but the true relationship is **convex**. Duration alone underestimates gains from falling rates and overestimates losses from rising rates. Convexity corrects this.

Because of convexity, a bond gains more in price on a drop in yield than it loses on an equal rise in yield.

LOS A · CALCULATING CONVEXITY

Approximate Convexity

$$ApproxConvexity = (PV_{-} + PV_{+} - 2 \times PV_0)/[(\Delta y)^2 \times PV_0]$$

What determines convexity?

A fixed-rate bond has **greater convexity** when:

- the longer its time to maturity;
- the lower its coupon;
- the lower its YTM.

Anything that leads to a higher ModDur also leads to higher convexity.

LOS B · DURATION + CONVEXITY FORMULA

Percentage Price Change with Duration *and* Convexity

EXAMPLE **Total price change, 50 bp yield drop**

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AnnModDur = 8.010, AnnConvexity = 75.270, ΔYTM = –0.005 (50 bp decrease).

SOLUTION

Convexity adds roughly 9 bp to the duration estimate of +4.005%.

CHECKPOINT 1

Quiz 1: Convexity Basics

Answer all questions to test your understanding. Your score updates on the final step.

Q1. The convexity adjustment for an **option-free fixed-rate bond** is:

Q2. A bond has AnnModDur = 8.010 and AnnConvexity = 75.270. For a 50 bp **rise** in YTM, the estimated %-price change is *closest* to:

Q3. Which change would *most likely* increase a bond's convexity?

Q4. Because of convexity, a 100 bp decrease in YTM produces a price increase that is:

Q5. ApproxConvexity is calculated as:

LOS B · MONEY CONVEXITY

Money Convexity

As with money duration, the money (dollar) version of convexity expresses the price change in currency units.

$$\text{MoneyConvexity} = \text{AnnConvexity} \times PV_{full}$$
$$\Delta PV_{full} = (-\text{MoneyDur} \times \Delta y) + (\tfrac{1}{2} \times \text{MoneyConvexity} \times \Delta y^2)$$

LOS C · PORTFOLIO DURATION METHODS

Two Ways to Compute Portfolio Duration

METHOD	WEIGHTED-AVERAGE TIME TO RECEIPT OF AGGREGATE CFS	WEIGHTED-AVERAGE OF INDIVIDUAL BOND DURATIONS
Description	Treat the portfolio like one big bond using its cash-flow yield (IRR on the aggregate CFs)	Weights = full price of each bond / total portfolio value; sum weight × bond duration
Practicality	Theoretically correct but rarely used	**Commonly used**
Cash-flow yield	Required (not commonly known)	Not required
Embedded options	Cannot use (uncertain CFs)	Works with effective duration inputs
Key assumption	Change in CF yield ≠ change in YTMs of individual bonds	**Parallel shift** in the yield curve

$$D_{port} = \sum w_i \times D_i \quad (where w_i = PV_i, full / \sum PV_j, full)$$

Limitations

- The weighted-average method **assumes parallel shifts** in the yield curve, which seldom happens in reality.
- The aggregate-CF-yield method ignores that individual bond YTMs may move differently from the cash-flow yield.

CHECKPOINT 2

Quiz 2: Money Convexity & Portfolio

Answer all questions to test your understanding. Your score updates on the final step.

Q6. The **weighted-average** approach to portfolio duration:

Q7. A portfolio holds $100 of Bond A (ModDur = 5) and 100 of Bond B (ModDur = 10). Its duration (weighted average) is:

Q8. Money convexity equals:

Q9. The theoretically correct portfolio-duration method (time to receipt of aggregate CFs) is rarely used because:

Q10. Empirically, if a portfolio's AnnConvexity is 35 and its AnnModDur is 4.8, the convexity adjustment for a 100 bp increase in YTM is:

LOS C · LIMITATIONS

What Duration Can’t Tell You

- Modified duration is a **linear estimate**; accuracy degrades for large yield changes. Convexity fixes this partially.
- Modified duration assumes **fixed cash flows**. For callable / putable / MBS, use **effective duration**.
- Portfolio duration via weighted average assumes **parallel shifts**. Key-rate durations (LM13) measure sensitivity to specific points on the curve.
- Even convexity corrections do not account for **credit-spread changes** — empirical duration handles that.

CHECKPOINT 3 · SYNTHESIS

Quiz 3: Synthesis

Answer all questions to test your understanding. Your score updates on the final step.

Q11. A bond’s AnnModDur is 7.25 and AnnConvexity is 58. For a 75 bp decline in YTM, the estimated %-price change is *closest* to:

Q12. Convexity of a 10-year zero-coupon bond, compared with a 10-year 5% coupon bond of equal YTM:

Q13. The main limitation of the **weighted-average portfolio duration** method is that it:

Q14. If rates fall sharply and yields turn volatile, ignoring convexity in a long-duration bond portfolio will:

Q15. A fixed-rate bond’s convexity is **largest** when the bond has:

MODULE COMPLETE

Your Scorecard

0 / 15

LM12 Key Takeaways — Exam-Ready Cheat Sheet

Price change formula

ApproxConvexity

$$(PV_- + PV_+ - 2PV_0)/[(\Delta y)^2 \times PV_0]$$

Convexity is larger for

Longer maturity · lower coupon · lower YTM. Same factors as duration.

Portfolio methods

Aggregate CFs (theoretical, rare) vs. **weighted average of individual durations** (common; assumes parallel shifts).

Step 1 of 11