LM13 · FIXED INCOME · CFA LEVEL I

Curve-Based and Empirical Fixed-Income Risk Measures

Effective duration & convexity for bonds with uncertain cash flows, key-rate duration for non-parallel shifts, and empirical vs analytical duration.

Learning Outcomes

LOS a Calculate and interpret effective duration and effective convexity for callable / putable bonds

LOS b Define, calculate, and interpret key-rate duration

LOS c Explain the difference between empirical and analytical duration

Exam Tip: Modified duration uses ΔYTM in the denominator; **effective duration uses Δ(benchmark curve)**. Use effective duration for any bond with embedded options, MBS, or uncertain CFs.

LOS A · EFFECTIVE DURATION

Effective Duration

When cash flows may change with rates (callables, putables, MBS), we cannot use YTM in the denominator. Instead we shock the **benchmark yield curve**.

$$EffDur = (PV_- - PV_+)/(2 \times \Delta curve \times PV_0)$$

Effective Convexity

$$EffCon = (PV_- + PV_+ - 2 \times PV_0)/[(\Delta curve)^2 \times PV_0]$$

When to use it: callable, putable, or MBS bonds — cash flows change with rates, so modified duration is inappropriate.

LOS A · CALLABLE BOND CONVEXITY

Why Callables Can Have *Negative* Convexity

For callable bonds, the issuer’s call option **caps** price appreciation. Near the call price, a further rate cut triggers a call instead of a price rise — the bond exhibits **negative convexity**.

Callable bond

Price is capped at or near the call price. For a bond with negative convexity, the price **increase** from a ↓YTM is **smaller** than the price **decrease** from a matching ↑YTM.

Putable bond

Always has **positive convexity**. At higher yields, the put is more valuable, so price declines less than that of an option-free bond. Duration of a putable < equivalent option-free at high yields.

Key insight: one-sided-effect (OSE) durations are also used for callables — EffDur on a rate **cut** can be much smaller than on a rate **rise**.

CHECKPOINT 1

Quiz 1: Effective Measures

Answer all questions to test your understanding. Your score updates on the final step.

Q1. For a **callable bond**, which duration measure is most appropriate?

Q2. A callable bond **can** exhibit *negative convexity* when:

Q3. A putable bond always exhibits:

Q4. Effective duration is calculated using:

Q5. At high yields, a putable bond’s **duration** is:

LOS B · KEY-RATE (PARTIAL) DURATION

Key-Rate Duration

A **key-rate duration** measures sensitivity of a bond (or portfolio) to a shock at *one specific point* on the spot-rate curve, holding other rates constant. It isolates exposure to *non-parallel* shifts (steepening, flattening, twists).

$$KeyRateDur_i = ModDur_{bondi} \times (Positionsize_i/Totalportfoliosize)Portfolioduration = \sum$$

EXAMPLE

Three-bond portfolio, total PV = 263

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Bond 2yr PV = 98.03, ModDur = 1.980; Bond 5yr PV = 90.57, ModDur = 4.902; Bond 10yr PV = 74.40, ModDur = 9.709. Total portfolio PV = 263.

SOLUTION

$$KeyRateDur_2 = 1.980 \times (98.03/263) = \mathbf{0.738}KeyRateDur_5 = 4.902 \times (90.57/263$$

TAKEAWAY

The sum of key-rate durations equals the total portfolio duration. Each tells you how much the portfolio moves if only the 2-yr / 5-yr / 10-yr rate moves.

LOS C · EMPIRICAL VS ANALYTICAL DURATION

Empirical vs Analytical Duration

Analytical duration

Uses mathematical formulas to estimate duration and convexity. Implicitly assumes benchmark yields and credit spreads are **uncorrelated**.

Empirical duration

Uses statistical methods and **historical bond prices** to derive price-yield sensitivity. Does not rely on the uncorrelated-spread assumption.

- For **government bonds**, both approaches give similar results.
- For **risky corporate bonds**, a “flight to quality” can cause spreads to widen when government yields fall → corporate prices rise less, or fall. Empirical duration captures this; analytical duration doesn’t.
- Empirical duration is usually **lower** than analytical duration for risky issues.

CHECKPOINT 2

Quiz 2: Key Rate & Empirical

Answer all questions to test your understanding. Your score updates on the final step.

Q6. Key-rate duration is *most useful* for:

Q7. A portfolio has key-rate durations of 0.738 (2-yr), 1.688 (5-yr), and 2.747 (10-yr). Total portfolio duration is:

Q8. Empirical duration of a corporate bond portfolio in a **flight-to-quality** episode is:

Q9. The primary drawback of **analytical duration** for a risky corporate bond is that it:

Q10. Effective convexity for a putable bond is:

REVIEW · WHEN TO USE EACH DURATION

Duration Decision Tree

BOND TYPE	PRIMARY RISK DRIVER	APPROPRIATE DURATION MEASURE
Option-free fixed-rate	ΔYTM	Modified duration
Callable / putable / MBS	Δbenchmark curve with CF changes	Effective duration
Portfolio, non-parallel shifts	Twists of curve	Key-rate duration
Risky corporate (flight to quality)	Correlated benchmark & spread moves	Empirical duration

CHECKPOINT 3 · SYNTHESIS

Quiz 3: Synthesis

Answer all questions to test your understanding. Your score updates on the final step.

Q11. A mortgage-backed security's price sensitivity is *most accurately* captured by:

Q12. Between two otherwise identical bonds, the one with the *lower* effective duration at high yields is:

Q13. Key-rate durations are particularly valuable when:

Q14. For most government bonds, analytical and empirical duration:

Q15. A bond portfolio's weighted-average duration assumes:

MODULE COMPLETE

Your Scorecard

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LM13 Key Takeaways — Exam-Ready Cheat Sheet

Effective measures

$$EffDur = (PV_- - PV_+)/(2 \times \Delta curve \times PV_0) EffCon = (PV_- + PV_+ - 2PV_0)/[(\Delta cu$$

Key-rate duration

Sum of KRDs = total duration. KRD_i = $ModDur_i$ × (PV_i / PV_{port}). Captures non-parallel shifts.

Negative convexity

Callables → may be negative at low yields (price capped near call). Putables → always positive.

Empirical vs analytical

Empirical duration uses actual price-yield data → captures spread-yield correlation. For risky corporates in a flight-to-quality event, empirical < analytical.

Step 1 of 10