LM02 · PORTFOLIO MANAGEMENT · CFA LEVEL I

# Portfolio Risk and Return: Part II

From CAL to CML, systematic vs. unsystematic risk, beta, CAPM, SML, and performance ratios

### What This Reading Covers

Part I ended with the Capital Allocation Line. Part II adds two key assumptions — **homogeneous expectations** and **informational efficiency** — which collapse all optimal risky portfolios to a single **market portfolio**, giving us the **Capital Market Line (CML)**, the **CAPM**, and the **Security Market Line (SML)**. It then distinguishes systematic from unsystematic risk, defines beta, and introduces performance-evaluation ratios.

## Learning Outcome Statements

| | |
|---|---|
| **LOS a** | Describe implications of combining a risk-free asset with a portfolio of risky assets |
| **LOS b** | Explain the CAL and the CML |
| **LOS c** | Explain systematic and nonsystematic risk, including why no reward for nonsystematic risk |
| **LOS d** | Explain return generating models (including the market model) and their uses |
| **LOS e** | Calculate and interpret beta |
| **LOS f** | Explain the CAPM, its assumptions, and the SML |
| **LOS g** | Calculate and interpret expected return of an asset using the CAPM |
| **LOS h** | Describe and demonstrate applications of the CAPM and SML |
| **LOS i** | Calculate and interpret Sharpe ratio, Treynor ratio, $M^2$, and Jensen's alpha |

**Exam Tip:** CAPM formula, SML vs CML difference, β calculation, Jensen's alpha, and the ranking of the four performance ratios are the absolute must-knows.

LOS A, B · CAPITAL MARKET LINE

# From CAL to CML

Multiple portfolios A, B, C on the Markowitz efficient frontier generate multiple CAL(A), CAL(B), CAL(C). The **optimal CAL** is the one with the **highest Sharpe ratio** (steepest slope) — in Mark Meldrum's notation, CAL(C).

## Two Additional Assumptions

HOMOGENEITY OF EXPECTATIONS

All investors have **the same** economic expectations regarding risk-return distributions for each asset.

⇒ only one optimal risky portfolio, **the market portfolio M**.

INFORMATIONALLY EFFICIENT MARKETS

No excess return from active investing. If not, active management could deliver alpha.

**Definition: Capital Market Line (CML)**

A CAL where the risky portfolio *is* the market portfolio M. M is the optimal risky-asset portfolio given homogeneous expectations; it typically includes all investable, tradable assets (in practice: a country's major equity index).

$$E(R_P) = R_f + [(E(R_m) - R_f)/\sigma_m] \cdot \sigma_P$$

The slope is again the **market price of risk**.

WORKED EXAMPLE **Lending and Borrowing on the CML**

GIVEN

$R_f$ = 5%, $R_m$ = 15%, $\sigma_m$ = 20%. Slope = (0.15 − 0.05)/0.20 = **0.50**. Every unit of σ delivers 0.5 units of excess return on the CML.

LENDING PORTFOLIOS (MIX $R_F$ + M)

| $W_1$ ($R_F$) | $W_2$ (M) | E($R_P$) | $Σ_P$ |
| --- | --- | --- | --- |

| | | | |
|---|---|---|---|
| 100% | 0% | 5% | 0% |
| 75% | 25% | 7.5% | 5% |
| 50% | 50% | 10% | 10% |
| 25% | 75% | 12.5% | 15% |
| 0% | 100% | 15% | 20% |

### BORROWING PORTFOLIOS (LEVERAGE M)

| LEVERAGE | $W_1$ | $W_2$ | $E(R_P)$ | $\Sigma_P$ |
|---|---|---|---|---|
| 25% | −0.25 | 1.25 | 17.5% | 25% |
| 50% | −0.50 | 1.50 | 20% | 30% |
| 100% | −1.00 | 2.00 | 25% | 40% |

### BORROWING AT A HIGHER RATE

If lending $R_f$ = 5% but borrowing $R_b$ = 7%, the CML kinks at M: slope becomes (0.15 − 0.07)/0.20 = 0.40 above M. Example: borrow 75% ⇒ $w_1$ = −0.75, $w_2$ = 1.75 ⇒ $E(R_P)$ = (−0.75)(0.07) + 1.75(0.15) = **21%**, $\sigma_P$ = (1 − (−0.75))(0.20) = **35%**.

LOS C · TWO TYPES OF RISK

# Systematic vs. Nonsystematic Risk

$$Total variance = Nonsystematic variance + Systematic variance$$

NONSYSTEMATIC RISK

**Diversifiable · firm-specific · unique** — pertains to a single company or industry. Eliminated via a portfolio of assets not highly correlated with each other.

SYSTEMATIC RISK

**Nondiversifiable · market risk** — affects the entire market/economy. Examples: interest rates, inflation, economic cycles, geopolitical uncertainty.

### Why Nonsystematic Risk Is Not Rewarded

Suppose investors *were* rewarded for both kinds of risk. They would buy assets with large nonsystematic risk, diversify it away — **get paid for risk they can avoid**. All investors would do this. Demand for diversifiable-risk assets would drive up their price, thereby reducing potential return. Conclusion: **no incremental reward can be earned for taking diversifiable risk**.

### Capital Market Theory — Core Rule

The market expects a **higher return on the investment with higher systematic risk**, *regardless of total risk*. Nonsystematic risk is not rewarded in an efficient market.

## Quick Check — Describe the risks of:

- **3-month T-bill:** essentially zero systematic risk; treated as the risk-free asset.
- **S&P 500 with σ = 20%:** all systematic — by definition it represents market risk.
- **Single-stock portfolio with σ = 30% vs. σ = 17%:** the one with more *systematic* risk should command higher E(R) — total σ is not what matters.

LOS D · MULTI-FACTOR AND SINGLE-INDEX MODELS

# Return Generating Models

If only systematic risk is rewarded, we need a way to determine what $E(R_i)$ should be. Return-generating models do this.

### Multi-Factor Model (general form)

$$E(R_i) - R_f = \beta_{i1} \cdot [E(R_m) - R_f] + \Sigma_{k=2}^{K} \beta_{ik} \cdot E(F_k)$$

**Macroeconomic**
GDP growth, interest rates, inflation

**Fundamental**
Earnings, cash flows, size, value

**Statistical**
Principal components, factor extraction

Note: research finds little evidence that the second-term factors add much beyond the single market factor.

### Single-Index Model / Market Model

$$E(R_i) - R_f = \beta_i \cdot [E(R_m) - R_f] \quad \rightarrow \quad E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

Move from expectations to actual data:

$$R_i = \alpha_i + \beta_i \cdot R_m + e_i$$

$\alpha_i$ and $\beta_i$ are estimated from **historical security-market return regressions**.

WORKED EXAMPLE **Walmart Market-Model Decomposition**

GIVEN

Daily-return regression of WMT on S&P 500 gives $\alpha = 0.0001$, $\beta = 0.9$. Today $R_m = 1\%$, observed $R_{WMT} = 2\%$.

PREDICTED RETURN FROM MARKET FACTOR

$\hat{R}_{WMT} = \alpha + \beta \cdot R_m = 0.0001 + 0.9(0.01) = 0.0091$ or 0.91%

**RESIDUAL (NONSYSTEMATIC) PORTION**

Nonsys return = $R_{WMT} - \hat{R} = 0.02 - 0.0091 =$ **0.0109 or 1.09%**

LOS E · BETA CALCULATION

# Beta — Measure of Systematic Risk

Beta measures how sensitive an asset's return is to the market as a whole — captures an asset's systematic risk. The average β of stocks in the market must be **1.0**. Most stocks have β ≥ 0.5.

$$\beta_i = \text{Cov}(R_i, R_m)/\sigma_m^2 = \rho_{im} \cdot (\sigma_i/\sigma_m)$$

WORKED EXAMPLE **Beta of Four Assets**

GIVEN $\Sigma_M$ = 25%

| ASSET | $\Sigma_I$ | $P_{IM}$ | $B = P \cdot (\Sigma/\Sigma_M)$ |
|---|---|---|---|
| 30-day T-Bill | ≈0 | — | **0** |
| Gold | 25% | 0 | **0** |
| Asset 3 | 60% | −0.1 | −0.1·(0.60/0.25) = **−0.24** |
| Asset 4 | 40% | 0.7 | 0.7·(0.40/0.25) = **1.12** |

TAKEAWAY

Gold at σ = 25% (same as market) has β = 0 because it's uncorrelated. Asset 3 has negative β (hedges market). Asset 4 is more volatile than the market.

## Practical β — Market Model Regression

Rather than computing $\sigma_i$, $\rho_{im}$, and $\sigma_m$ separately, analysts regress $R_i$ on $R_m$:

$$R_i = \alpha_i + \beta_i \cdot R_m + e_i$$

LOS F, G · CAPM

# CAPM and the Security Market Line (SML)

$$E(R_i) = R_f + \beta_i \cdot [E(R_m) - R_f]$$

## Six CAPM Assumptions

1. Investors are **utility maximizing, risk-averse, rational**.
2. Markets are **frictionless** — no transaction costs, no taxes, borrowing & lending at $R_f$.
3. All investors have the **same single-period investment horizon**.
4. Investors have **homogeneous expectations** — same valuation for any given asset.
5. All investments are **infinitely divisible**.
6. Investors are **price takers** — no investor large enough to influence prices.

## CAL/CML vs. SML

| | CAL / CML | SML |
|---|---|---|
| X-axis | **σ (total risk)** | **β (systematic risk)** |
| Plots | Efficient portfolios only | All securities AND all portfolios |
| Slope | $(E(R_m)-R_f) / \sigma_m$ (Sharpe) | $(E(R_m)-R_f)$ market risk premium |
| Applies to | Well-diversified portfolios | Individual securities too (β captures only systematic risk) |

**Exam Tip:** CAL/CML plot σ; SML plots β. CML slope = Sharpe; SML slope = market risk premium $(E(R_m) - R_f)$.

LOS G · EXPECTED RETURN VIA CAPM

# Applying the CAPM

WORKED EXAMPLE **CAPM Portfolio β and E(R)**

### GIVEN

| W | ASSET |
|---|---|
| 20% | 30-day T-Bill @ 4% |
| 30% | S&P 500, $E(R_m) = 16\%$ |
| 50% | U.S. stock with $\beta = 2.0$ |

### PORTFOLIO Β

$\beta_P = 0.2(0) + 0.3(1) + 0.5(2) = 0 + 0.3 + 1.0 = \mathbf{1.3}$

### PORTFOLIO E(R) VIA CAPM

$E(R_P) = R_f + \beta_P \cdot (E(R_m) - R_f) = 0.04 + 1.3(0.16 - 0.04) = 0.04 + 1.3(0.12) = \mathbf{19.6\%}$

### PROOF VIA WEIGHTED INDIVIDUAL $E(R_I)$

$E(R_{Tbill}) = 4\%$, $E(R_m) = 16\%$, $E(R_{stock}) = 0.04 + 2(0.12) = 28\%$.

Weighted: $0.2(4) + 0.3(16) + 0.5(28) = 0.8 + 4.8 + 14 =$ **19.6%** ✓

QUIZ 1 · CHECKPOINT

## Test Your Understanding: CML, Risk, CAPM

**Q1.** The Capital Market Line (CML) differs from a generic Capital Allocation Line (CAL) in that:

**Q2.** Why does efficient-market theory say investors are *not* rewarded for bearing nonsystematic risk?

**Q3.** Asset X has $\sigma = 60\%$, correlation with market = −0.1, and market $\sigma = 25\%$. Its $\beta$ is *closest to*:

**Q4.** Using CAPM with $R_f = 3\%$, $E(R_m) = 9\%$, and $\beta = 1.5$, E(R) is:

**Q5.** Which is *not* a CAPM assumption?

LOS H · USING THE CAPM

# CAPM Applications

## ① Estimate Required Return → Buy/Hold/Sell

**WORKED EXAMPLE** **Stock Valuation: Buy / Hold / Sell**

**GIVEN**

$P_0$ = \$20.07, β = 1.15, $D_0$ = \$1.22, 30-day T-bill = 2.1%, g = 6.1%, ERP = 8.4%. Threshold ±5% for Buy/Sell decision.

**REQUIRED RETURN (CAPM)**

E(R) = $R_f$ + β·ERP = 0.021 + 1.15(0.084) = **11.76%**

**FAIR VALUE VIA GORDON GROWTH**

$P_0^*$ = $D_0$(1+g) / (r − g) = 1.22(1.061) / (0.1176 − 0.061) = 1.2944 / 0.0566 = **\$22.87**

**DECISION**

Actual price $20.07 vs. intrinsic$22.87 ⇒ undervalued by ~14% (well outside ±5% threshold). **BUY.**

## ② Capital Budgeting (Project Discount Rate)

**WORKED EXAMPLE** **NPV Using Project β**

**GIVEN**

$R_m$ = 12%, $R_f$ = 2%, project β = 2.3.

**REQUIRED RETURN**

E(R) = 0.02 + 2.3(0.12 − 0.02) = **25%**

**NPV (FROM MM SOURCE)**

CFs: Yr1 −500M, Yr2 −200M, Yr3 +400M, Yr4 +400M, Yr5 +400M, Yr6 +1000M. Discount at 25% ⇒ **NPV = −$147.07M (reject)**.

LOS I · SHARPE, TREYNOR, M², JENSEN'S A

# Portfolio Performance Evaluation

| MEASURE | FORMULA | RISK USED | USE WHEN |
|---|---|---|---|
| **Sharpe Ratio** | $(R_P - R_f) / \sigma_P$ | Total risk σ | Any portfolio (efficient or not) |
| **Treynor Ratio** | $(R_P - R_f) / \beta_P$ | Systematic β | Fully diversified portfolios only |
| **M² (M-squared)** | $(R_P - R_f)\cdot(\sigma_m/\sigma_P) - (R_m - R_f)$ | Total risk σ | Same as Sharpe — percentage form |
| **Jensen's α** | $R_P - [R_f + \beta_P(R_m - R_f)]$ | Systematic β | Fully diversified; percentage excess return |

WORKED EXAMPLE **Comparing Four Managers**

GIVEN $R_F$ = 3%, $R_M$ = 9%, $Σ_M$ = 15%

| MGR | R | Σ | Β |
|---|---|---|---|
| X | 10% | 20% | 1.1 |
| Y | 11% | 10% | 0.7 |
| Z | 12% | 25% | 0.6 |
| Market M | 9% | 15% | 1.0 |

SHARPE RATIO

X = (0.10−0.03)/0.20 = 0.35
Y = (0.11−0.03)/0.10 = **0.80**
Z = (0.12−0.03)/0.25 = 0.36
M = (0.09−0.03)/0.15 = 0.40

TREYNOR RATIO

X = 0.07/1.1 = 0.0636

Y = 0.08/0.7 = 0.114

Z = 0.09/0.6 = **0.15**

M = 0.06/1.0 = 0.06

### JENSEN'S Α = R − [$R_F$ + Β($R_M$−$R_F$)]

Expected: X = 9.6%, Y = 7.2%, Z = 6.6%.

$\alpha_X$ = 10% − 9.6% = **+0.4%**

$\alpha_Y$ = 11% − 7.2% = **+3.8%**

$\alpha_Z$ = 12% − 6.6% = **+5.4%**

$\alpha_M$ = 0%

### RANKINGS

**Sharpe (total risk):** Y > Z > X > M (Y best on risk-adjusted total-risk basis)

**Treynor (systematic only):** Z > Y > X > M

Different rankings arise because Sharpe penalizes total σ (nonsystematic + systematic), while Treynor only penalizes β.

LOS I · SECURITY CHARACTERISTIC LINE & HETEROGENEOUS BELIEFS

# Security Characteristic Line & Security Selection

$$R_i - R_f = \alpha_i + \beta_i \cdot (R_m - R_f) + e_i$$

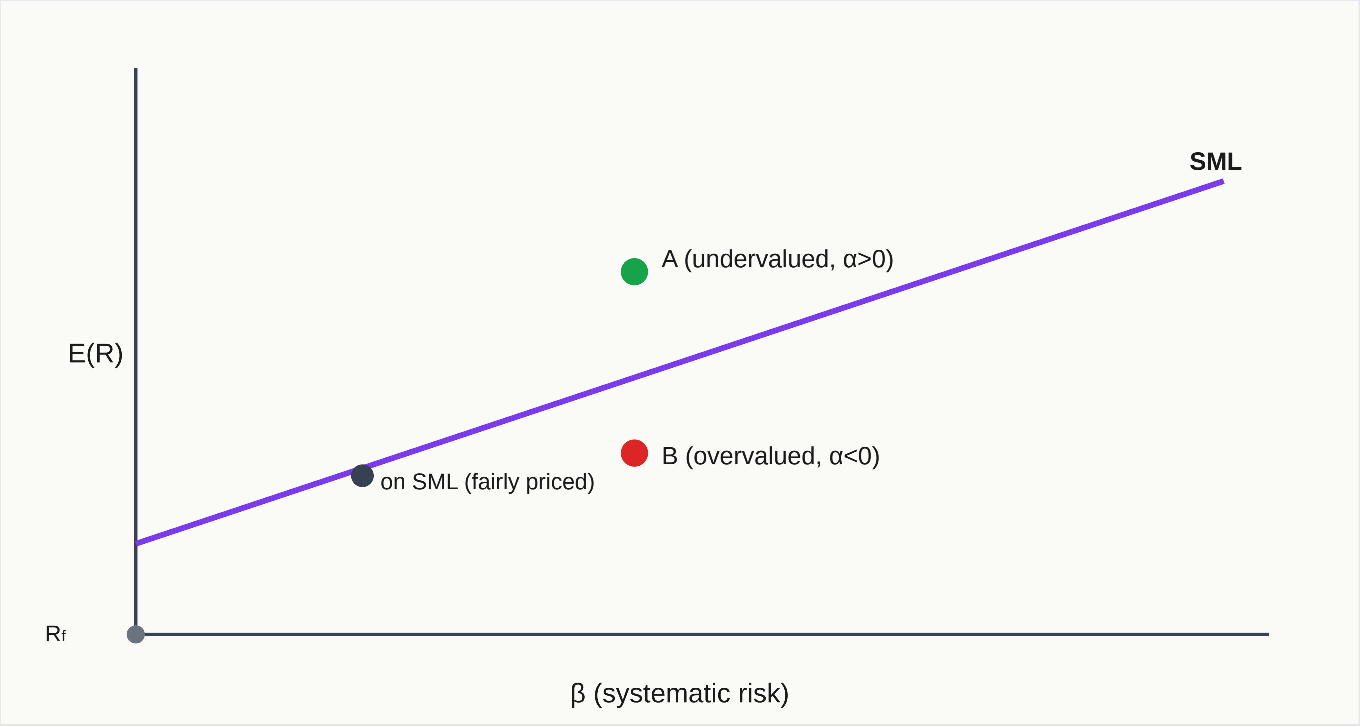
Rearranging: $\alpha_i$ (Jensen's α) is the intercept of excess returns — the **abnormal return** on the security after adjusting for systematic-risk exposure.

### Security-Selection Rules

- **Select / overweight** securities with $\alpha_i > 0$ (undervalued).
- **Deselect / underweight / short** securities with $\alpha_i < 0$ (overvalued).

## Heterogeneous Expectations

If all investors have *homogeneous* expectations, every security lies on the SML and α = 0. If beliefs differ (regarding ① future cash flows, ② systematic risk, or ③ both), some securities will appear *above* the consensus SML (undervalued) and some *below* (overvalued).



QUIZ 2 · CHECKPOINT

## Test Your Understanding: Beta, Models, Performance

**Q6.** A portfolio contains: 20% T-bills ($\beta$=0), 30% market index ($\beta$=1), 50% stock with $\beta$=2. Portfolio $\beta$ is:

**Q7.** For the same portfolio, if $R_f$ = 4% and $E(R_m)$ = 16%, $E(R_p)$ using CAPM is:

**Q8.** Manager Z has R = 12%, $\sigma$ = 25%, $\beta$ = 0.6. $R_f$ = 3%, $R_m$ = 9%. Z's Jensen's $\alpha$ is:

**Q9.** Two managers have the *same* Jensen's $\alpha$ but different Sharpe ratios. The difference is attributable to:

**Q10.** The Security Market Line is *best* described as:

LOS G · CAPM — Β SENSITIVITY

# How Changing β Affects Required Return

WORKED EXAMPLE

## CAPM Before / After β Change

### GIVEN

$R_f$ = 3%, $E(R_m)$ = 10%, Initial β = 1.5. What if β later drops to 1.1?

### BEFORE (Β = 1.5)

E(R) = 0.03 + 1.5(0.10 − 0.03) = 0.03 + 0.105 = **13.5%**

### AFTER (Β = 1.1)

E(R) = 0.03 + 1.1(0.10 − 0.03) = 0.03 + 0.077 = **10.7%**

### TAKEAWAY

Because both $R_f$ and $E(R_m)$ shift with economic conditions, the SML slope (market price of risk) and intercept both matter — but β determines an asset's vertical position on the line.

## Capital Market Theory — Review

Under the Markowitz framework we needed a **full covariance matrix** for all pairs of investments (n(n−1)/2 covariances). With the market model, we simplify dramatically — each security needs one α and one β vs. $R_m$. For n = 50 assets: **50 betas vs. 1,225 covariances**.

LOS I · COMPARING PERFORMANCE METRICS

# Comparing the Four Performance Metrics

| METRIC | RISK-ADJUSTED BASIS | BEST USED ON... | SHAPE |
|---|---|---|---|
| **Sharpe** | Total σ | Any portfolio | Slope (ratio) |
| **$M^2$** | Total σ | Any portfolio | Percentage excess return |
| **Treynor** | β (systematic) | Fully diversified only | Slope (ratio) |
| **Jensen's α** | β (systematic) | Fully diversified only | Percentage excess return |

### Why Rankings Can Differ

Sharpe and $M^2$ use **total risk**, so they penalize undiversified managers. Treynor and Jensen's α use only β — they ignore firm-specific risk. A skilled stock-picker with concentrated positions may have high Jensen's α but poor Sharpe.

**Exam Tip:** When in doubt: Sharpe for any portfolio, Jensen's α or Treynor for fully diversified portfolios that are part of a larger multi-manager framework.

QUIZ 3 · CHECKPOINT

## Test Your Mastery

**Q11.** A stock has $R_{WMT}$ = 2% today, α = 0.0001, β = 0.9, and $R_m$ = 1%. The component due to *nonsystematic* risk is *closest to*:

**Q12.** With $R_f$ = 5%, $R_m$ = 15%, $\sigma_m$ = 20%, and borrowing rate $R_b$ = 7%, an investor who borrows 75% at $R_b$ and holds 175% in M has $\sigma_P$ *closest to*:

**Q13.** A security plots *above* the SML. It is *most likely*:

**Q14.** The *key distinction* between the CML and the SML is:

**Q15.** For manager Y (R = 11%, σ = 10%, β = 0.7) with $R_f$ = 3%, $R_m$ = 9%, which ratio is *closest to* correct?

MODULE COMPLETE

## Your Scorecard

# 0 / 15

## LM02 Key Takeaways — Exam-Ready Cheat Sheet

### CML and Systematic Risk

1. **CML**: $E(R_P) = R_f + [(R_m−R_f)/\sigma_m]\cdot\sigma_P$. Derives from homogeneity + efficient markets.
2. **Total variance = systematic + nonsystematic.** Only systematic risk is rewarded.
3. If nonsystematic risk were rewarded, arbitrage would eliminate the reward.

### Beta, CAPM, SML

4. $\beta = Cov(R_i,R_m)/\sigma_m^2 = \rho\cdot(\sigma_i/\sigma_m)$. Market $\beta = 1$.
5. **CAPM**: $E(R_i) = R_f + \beta_i\cdot[E(R_m) − R_f]$. 6 assumptions.
6. **SML** plots CAPM in ($\beta$, E(R)) space — applies to all securities.
7. CML ($\sigma$-axis, efficient portfolios only) vs. SML ($\beta$-axis, everything).

### Performance Ratios

8. **Sharpe** = $(R−R_f)/\sigma$ · **M²** = percentage form of Sharpe. Use: any portfolio.
9. **Treynor** = $(R−R_f)/\beta$ · **Jensen's α** = $R − [R_f + \beta(R_m−R_f)]$. Use: fully diversified only.
10. Rankings differ when portfolios have different levels of nonsystematic risk.

## Last-Minute Exam Checklist

**CAPM:** $E(R_i) = R_f + \beta(MRP)$

**CAL/CML:** σ-axis, efficient only · **SML:** β-axis, every security

**β:** $\rho(\sigma_i/\sigma_m)$; market $\beta = 1.0$; T-bill $\beta = 0$

**Jensen's α > 0** = above SML = undervalued; < 0 = overvalued

**Sharpe/M² = total risk · Treynor/Jensen = systematic risk**

**6 CAPM assumptions:** rational investors · frictionless markets · same horizon · homogeneous expectations · divisibility · price takers

Step 1 of 16