LM01 · QUANTITATIVE METHODS · CFA LEVEL I

Rates and Returns

Interest rates, return measures across time periods, and leveraged/real return adjustments

Three Ways to Think About Interest Rates (r)

1. **Required rate of return** — what an investor demands to accept an investment; determines the FV of a PV (invest $100today, require$110 back in one year → r = 10%).
2. **Discount rate** — used to determine the PV of a FV ($110oneyearoutdiscountedat10$100 today).
3. **Opportunity cost** — value forgone (current consumption vs. saving). Spending \$100 today costs 10% forgone return.

Learning Outcome Statements

LOS a	Interpret interest rates as required rates of return, discount rates, or opportunity costs and explain an interest rate as the sum of a real risk-free rate plus premiums
LOS b	Calculate and interpret different approaches to return measurement over time
LOS c	Compare money-weighted and time-weighted rates of return and evaluate portfolio performance
LOS d	Calculate and interpret annualized return measures and continuously compounded returns
LOS e	Calculate and interpret major return measures (gross, net, pre-tax, after-tax, real, leveraged) and describe their appropriate uses

Exam Tip: Every Quant reading downstream uses these definitions. The interest rate decomposition (5 components), HPR, arithmetic vs. geometric vs. harmonic mean, MWR vs. TWR, and the leveraged-return formula are the highest-frequency testables.

LOS A · DETERMINANTS OF INTEREST RATES

Decomposing the Nominal Interest Rate

NOMINAL INTEREST RATE

$$r = r_f + \text{Inflation premium} + \text{Default risk premium} + \text{Liquidity premium} + \text{Maturity premium}$$

r	nominal interest rate	r_f	real risk-free rate	π^e	expected inflation premium
DRP	default risk premium	LP	liquidity premium	MP	maturity premium

COMPONENT	MEANING
Real risk-free rate (r_f)	Return on a zero-risk single-period investment, assuming no inflation.
Inflation premium (π^e)	Compensation for expected inflation over the holding period.
Default risk premium	Compensation for the risk the borrower fails to make promised payments (credit risk).
Liquidity premium	Compensation for the risk of receiving less than fair value if the investment must be sold quickly for cash.
Maturity premium	Compensation for the greater price sensitivity of longer-maturity debt to changes in rates.

Nominal Risk-Free Rate

$$(1+r) = (1+r_f)(1+\pi^e) \quad \text{or approximately} \quad r \approx r_f + \pi^e$$

All rates are quoted on an annual basis. E.g., a 3-month T-Bill at 4% earns 0.04/4 = 1% over 3 months.

WORKED EXAMPLE **Calculating the Nominal Rate on a Corporate Bond**

GIVEN

In country ABC: real risk-free rate = 4%, expected inflation = 3%. Company X issues a 5-year bond with default risk premium = 2%, liquidity premium = 1%, maturity premium = 1%.

FIND

The nominal interest rate on the bond.

ANSWER

r = 4 + 3 + 2 + 1 + 1 = **11%**

LOS B · HOLDING PERIOD RETURN AND MEAN RETURN MEASURES

Holding Period Return and the Means

Single-Period Holding Period Return (HPR)

HOLDING PERIOD RETURN

$$\text{HPR} = \frac{P_T - P_0 + I}{P_0} = \text{capital gain} + \text{income yield}$$

P_T	ending price	P_o	beginning price
I	income (dividend/coupon) received	HPR	holding period return

P_T = ending price, P_0 = beginning price, I = income earned over the period.

WORKED EXAMPLE **Single-Period HPR**

GIVEN

Stock price: $40atstartofperiod \rightarrow$44 at end of period. Dividend paid during period: $2.

SOLUTION

HPR = (44 − 40 + 2) / 40 = 6 / 40 = **15%**

Multi-Period HPR

$$R = (1 + \text{HPR}_1)(1 + \text{HPR}_2)(1 + \text{HPR}_3) \cdots (1 + \text{HPR}_T) - 1$$

Arithmetic Mean Return

$$R_A = \frac{1}{T}\sum_{t=1}^{T} R_t \quad \text{(simple average of returns)}$$

Appropriate with **single-period or cross-sectional** data. Describes a representative outcome; sensitive to outliers.

Geometric Mean Return

GEOMETRIC MEAN RETURN

$$R_G = [(1 + R_1)(1 + R_2) \cdots (1 + R_T)]^{1/T} - 1$$

R_G	geometric mean return (compound growth rate)	R_t	period-t return	T	number of periods

The compound growth rate on an investment. Appropriate with **time-series** data. Use to estimate expected return over **multiple periods**.

Key Relationship: $R_G \leq R_A$

Unless all observations are equal, $R_G < R_A$. As the variability in the data increases, the difference between R_G and R_A **widens**.

LOS B · HARMONIC, TRIMMED, AND WINSORIZED MEANS

Harmonic Mean and Outlier-Robust Measures

Harmonic Mean

$$\bar{X}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}}$$

Each observation's weight is **inversely proportional** to its magnitude. Reduces the effect of large outliers.

WORKED EXAMPLE **Dollar-Cost Averaging with Harmonic Mean**

GIVEN

An investor purchased $1,000 of stock A each month for three months at prices of $5, $6, and $7.

SOLUTION

Harmonic mean average purchase price = 3 / (1/5 + 1/6 + 1/7) = **$5.88**

PROOF

$1,000/5 = 200 sh; $1,000/6 = 166.67 sh; $1,000/7 = 142.86 sh. Total shares = 509.52. Avg cost = $3,000 / 509.52 = **$5.89** (matches harmonic mean, rounding).

Relationship Among the Three Means

$$R_A \times \bar{X}_H = R_G^2 \quad \text{(for positive values)}$$

Outlier Techniques

TECHNIQUE	WHAT IT DOES

Trimmed mean	Removes a stated % from both tails. E.g., 8% trimmed mean on 100 obs. removes 8 highest and 8 lowest; common with CPI computations.
Winsorized mean	Replaces extreme values with the cutoff value. E.g., for a 90% winsorized mean on 100 obs., replace the 8 lowest with obs. #9 and the 8 highest with obs. #92.

Exam Tip: When to use each mean: **Arithmetic** → single-period/cross-sectional. **Geometric** → time-series/compound growth. **Harmonic** → average *cost* for equal periodic dollar investments. **Trimmed/Winsorized** → outlier-contaminated data.

LOS C · MONEY-WEIGHTED VS. TIME-WEIGHTED RETURNS

Money-Weighted vs. Time-Weighted Returns

Money-Weighted Return (IRR on Cash Flows)

The money-weighted return (MWR) is the **IRR on the cash flows** of an investment. It accounts for the **timing and magnitude** of cash inflows and outflows.

WORKED EXAMPLE MWR for a Two-Stock Portfolio (from MM notes)

GIVEN

Purchase XYZ at t=0 for $20; purchaseABCatt = 1for$30 (XYZ pays $1dividend).Att =$ $2, sellbothfor$60 each; both paid $1 dividend.

CASH FLOWS

CF_0 = −$20; CF_1 = $-30+1$ = −$29; CF_2 = 2×$60 + 2$×1 = +$122. Solve: $122/(1+r)^2$ = 29/(1+r) + 20.

ANSWER

MWR = **17.91%** (IRR).

Time-Weighted Return (TWR = R_G of sub-period HPRs)

The TWR is the **compound growth rate of $1 invested in a portfolio** over the measurement period. It is **not affected** by the timing or magnitude of cash contributions or withdrawals, which makes it the **industry standard** for evaluating portfolio managers.

Steps: (1) Break the period into sub-periods at each significant cash-flow date. (2) Compute each sub-period HPR. (3) Take the geometric mean, annualized.

WORKED EXAMPLE TWR with a Mid-Period Deposit

GIVEN

Buy one share at $10(t = 0).Year1 : receive$1 dividend; add a second share purchased at $12.Year2 : receive$0.50 per share; sell both for \$13 each.

SUB-PERIOD HPRS

HPY_1 = (12 − 10 + 1) / 10 = **30%**

HPY_2 = (26 − 24 + 1) / 24 = **12.5%**

ANSWER

$(1 + TWR)^2$ = 1.30 × 1.125 ⇒ TWR = **20.93%**

MWR vs. TWR — Decision Framework

Use the **MWR** when the portfolio manager *controls* the timing/magnitude of cash flows. Use the **TWR** when the manager does not. Contributions made just before strong performance bias MWR *above* TWR; contributions made just before weak performance bias MWR *below* TWR.

QUIZ 1 · CHECKPOINT

Test Your Understanding: Rates and Basic Returns

Q1. An interest rate can be interpreted in all of the following ways *except*:

Q2. In country ABC, r_f = 4%, expected inflation = 3%, and a 5-year corporate bond has default, liquidity, and maturity premia of 2%, 1%, and 1% respectively. The required yield is *closest to*:

Q3. A mutual fund returned −6%, +8%, and +14% in three successive years. The *geometric* mean annual return is *closest to*:

Q4. An investor buys $1,000 of a stock each month for three months at prices of$5, $6, and$7. The average purchase price per share (harmonic mean) is *closest to*:

Q5. When evaluating an external fund manager who *does not* control the timing of client contributions, the *most appropriate* performance measure is:

LOS D · ANNUALIZED AND CONTINUOUSLY COMPOUNDED RETURNS

Annualizing Returns & Continuous Compounding

Annualized Return

ANNUALIZED RETURN

$$R_{\text{annual}} = (1 + R_{\text{period}})^c - 1$$

R_period	return earned in one subperiod	c	number of subperiods per year	R_annual	annualized (effective) return

FREQUENCY	FORMULA	PERIODS/YR
Daily	$(1 + R)^{365} - 1$	365
Weekly	$(1 + R)^{52} - 1$	52
Monthly	$(1 + R)^{12} - 1$	12
Quarterly	$(1 + R)^{4} - 1$	4
43-day period	$(1 + R_{43})^{365/43} - 1$	365/43
540-day period	$(1 + R_{540})^{365/540} - 1$	365/540

Annualizing can be misleading — it assumes the period return can be repeated continuously.

Continuously Compounded Return (r_c)

$$r_c = \ln\left(\frac{P_t}{P_0}\right) = \ln(1 + \text{HPR})$$

WORKED EXAMPLE **Continuous vs. Discrete**

GIVEN

HPR of a stock was 10% over one year. Find the continuously compounded equivalent.

SOLUTION

r_c = ln(1.10) = **0.0953 = 9.53%**. Keystrokes: 1.1 [ln].

REVERSE

Starting from r_c = 4.879% on a 100 →105 stock: r = $e^{0.04879}$ − 1 = **5%**.

Why Continuous Compounding Matters

Continuously compounded rates **are additive over time** ($r_{c,2} = r_{c,1a} + r_{c,1b}$), making them the natural choice for option pricing, Black-Scholes, and log-return time-series modeling.

LOS E · GROSS, NET, AFTER-TAX, AND REAL RETURNS

Gross, Net, Pre-Tax, After-Tax, and Real Returns

GROSS RETURN

Return **before** deductions for management expenses, custodial fees, or taxes, but **after** trading expenses. Best for evaluating a manager's investment skill.

NET RETURN

What the investor actually earns — gross return minus all fees and expenses. The relevant measure for the end investor.

Pre-tax and After-tax Nominal Returns

Default reporting is pre-tax. Each investor's marginal tax rate may differ. Components of return are often broken out — e.g., a reported 11% might split as 2% dividends, 1% interest, 4% realized gains, 4% unrealized gains — because each piece has a different tax rate.

$$\text{After-tax nominal return} = \text{Pre-tax nominal return} \times (1 - \text{tax rate})$$

Real Returns

$$(1 + \text{nominal}) = (1 + \text{real}) \times (1 + \text{inflation})$$

Equivalently: 1 + nominal return = (1 + r_f) × (1 + risk premium) × (1 + inflation).

After-tax real return is the investor's measure of growth in *purchasing power* of their portfolio.

WORKED EXAMPLE **Approximate vs. Exact Real Return (Schweser)**

GIVEN

Nominal return = 7%, inflation = 2%.

SOLUTION

Approximate real return: 7% − 2% = **5%**.

Exact real return: 1.07 / 1.02 − 1 = **4.902%**.

LOS E · LEVERAGED RETURN

Leveraged Return

Leverage may be obtained through margin loans, derivatives, or collateralized loans (repos). If the unlevered portfolio return R_P exceeds the cost of borrowing r_d, leverage **enhances** return; if $R_P < r_d$, leverage **magnifies losses**.

LEVERAGED RETURN

$$R_L = R_P + \frac{V_B}{V_E} \times (R_P - r_d)$$

R_L	levered return on equity	*R_P*	unlevered portfolio return	*V_B*	value borrowed (debt)
V_E	equity contributed	*r_d*	cost of borrowing		

where V_B = value borrowed, V_E = equity, r_d = cost of borrowings, R_P = unlevered portfolio return.

WORKED EXAMPLE **Levering a 10M Portfolio (MM notes)**

GIVEN

$10M equity portfolio, R_P = 8%; 30% debt-financed (so $7Mequity/$3M borrowed), r_d = 5%.

SOLUTION

R_L = 8% + (3M / 7M) × (8% − 5%) = 8% + 0.4286 × 3% = **9.2857%**

Derivation — Why the Formula Works

Total portfolio gross return = $R_P \times (V_E + V_B)$. Interest cost = $V_B \times r_d$. Net profit to equity = $R_P(V_E + V_B) - V_B r_d$. Divide by V_E to get the levered return on equity: $R_P + (V_B/V_E)(R_P - r_d)$.

QUIZ 2 · CHECKPOINT

Test Your Understanding: Annualizing, Compounding, Return Types

Q6. A portfolio earned 3% over a 60-day period. Assuming 365 days in a year, the annualized return is *closest to*:

Q7. A stock's holding-period return was 10% for the year. The equivalent continuously compounded annual return is *closest to*:

Q8. If nominal return = 7% and inflation = 2%, the *exact* real return is *closest to*:

Q9. Which return is the *best* measure of a portfolio manager's investment skill?

Q10. An investor with a $7Mequitystakeleversaportfoliobyborrowing$3M at r_d = 5%. If the unlevered return is R_P = 8%, the levered return is *closest to*:

INTEGRATION · PUTTING IT ALL TOGETHER

An Investor's Full Return Decomposition

A global-equity investor holds a portfolio over one year and ends with a **gross nominal return of 11%**, composed of 2% dividends, 1% interest on cash, 4% realized gains, and 4% unrealized gains. Advisory/admin fees total 100 bps. Marginal tax rate on ordinary income is 30%, on realized capital gains 15%. Inflation during the year is 2.5%.

STAGE	CALCULATION	RESULT
Gross return (pre-fee)	—	11.00%
Net return (after 100 bps fee)	11% − 1%	10.00%
After-tax return on 3% ord. income	3% × (1 − 0.30)	2.10%
After-tax return on 4% realized gain	4% × (1 − 0.15)	3.40%
After-tax unrealized gain (deferred)	4.00%	4.00%
Net after-tax nominal (approx.)	2.10% + 3.40% + 4.00% − 1% fees	≈ 8.50%
After-tax real return	1.085 / 1.025 − 1	**≈ 5.85%**

Read This Carefully

The final **after-tax real return** measures the growth in the investor's *purchasing power*. This is the number that matters for long-horizon planning (retirement, college funding).

QUIZ 3 · CHECKPOINT

Test Your Understanding: Integration & Edge Cases

Q11. An investor purchases $100onday1, receives$3 in dividends during the year, and the stock is worth $103 at year-end. The HPR is:

Q12. If an investor contributes funds to a portfolio just *before* a period of relatively poor performance, then:

Q13. An analyst observes returns of −50%, +35%, and +27% for an investment over three years. The geometric mean return is *closest to*:

Q14. Which statement about trimmed and winsorized means is *most accurate*?

Q15. An investor lever a portfolio with 30% debt at r_d = 5%. If the unlevered return is 4%, what happens to the levered return?

MODULE COMPLETE

Your Scorecard

0 / 15

LM01 Key Takeaways — Exam-Ready Cheat Sheet

Interest Rate Components (memorize)

$$r = r_f + \pi^e + \text{default} + \text{liquidity} + \text{maturity}$$

Return Measures at a Glance

HPR	$(P_T - P_0 + I) / P_0$
Arithmetic	Single-period / cross-sectional; $R_A \geq R_G$
Geometric	Time-series / multi-period compound growth
Harmonic	Equal-dollar periodic investment (dollar-cost averaging)
Trimmed	Delete outliers on both tails
Winsorized	Replace outliers with cutoff values
MWR	IRR on cash flows; use when manager controls timing
TWR	Geometric mean of sub-period HPRs; industry standard
Annualized	$(1 + R_{period})^{periods/yr} - 1$
Continuous	$r_c = \ln(1 + HPR)$; additive over time

Gross, Net, Real, Leveraged

$$\text{After-tax nominal} = \text{Pre-tax} \times (1 - \text{tax})$$
$$(1 + \text{nominal}) = (1 + \text{real})(1 + \text{inflation})$$
$$R_L = R_P + \frac{V_B}{V_E}(R_P - r_d)$$

Last-Minute Exam Checklist

Rate components: r_f · inflation · default · liquidity · maturity

Means: AM (cross-section) · GM (time series) · HM (dollar-cost averaging)

$R_G \leq R_A$ · equality only if all values equal

MWR → IRR with CFs · **TWR** → $\prod(1 + HPR_i) - 1$

$r_c = \ln(1 + r)$ · always < r for positive returns

Real return: 1.07 / 1.02 − 1 = 4.90% (exact) vs. 5% (approx)

Leverage: only helps if $R_P > r_d$

Step 1 of 13